Math2210 Quiz 8 (Sections 12.9-13.1) Summer, 2010 Dylan Zwick

Name Solutions Date 7/27/2010

Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. *(15 Points) Find the maximum of* $f(x,y)=4x^2-4xy+y^2$ subject to the constraint $x^2+y^2=1$.

$\nabla f(x,y) = \langle 8x-4y,\ 2y-4x \rangle$ $g(x,y)=x^2+y^2-1$

$\nabla g(x,y) = \langle 2x,\ 2y \rangle$

$\nabla f = \lambda \nabla g \Rightarrow 8x-4y = 2\lambda x$
$2y-4x = 2\lambda y$

$\Rightarrow 4x-2y = \lambda x$
$-4x+2y = 2\lambda y$ $\Rightarrow 0 = \lambda(x+2y)$

Either $\lambda = 0$ in which case $y=2x$ and $x^2+y^2=1$

$\Rightarrow x^2+(2x)^2=1$

$\Rightarrow 5x^2=1$

$x^2=\frac{1}{5}$

$x=\pm\frac{1}{\sqrt{5}}$

Points

$\left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$

$\left(-\frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}}\right)$

$f\left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right) = f\left(-\frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}}\right)$

$= \frac{4}{5} - \frac{8}{5} + \frac{4}{5}$

$= 0.$

Minimum.

or $x=-2y$ and $x^2+y^2=1$

$\Rightarrow (-2y)^2+y^2=1 \Rightarrow 5y^2=1$

$y=\pm\frac{1}{\sqrt{5}}$

Points

$\left(-\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$

$\left(\frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}}\right)$

$f\left(-\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right) = f\left(\frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}}\right)$

$= \frac{16}{5} - 4\left(\frac{-2}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{5}}\right) + \frac{1}{5} = \frac{25}{5} = 5$

Maximum.

Answer: $\boxed{5}$ at $\left(-\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$ or $\left(\frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}}\right)$.

2. *(15 Points) Find the minimum distance between the origin and the plane* $x+3y-2z=4$.

$d=\sqrt{x^2+y^2+z^2}$ $\qquad \nabla f = \langle 2x, 2y, 2z\rangle$

$d^2 = x^2+y^2+z^2 = f(x,y,z)$ $\qquad \nabla g = \langle 1, 3, -2\rangle$

$g(x,y,z) = x+3y-2z-4$

$\nabla f = \lambda \nabla g \Rightarrow \quad 2x=\lambda \quad 2y=3\lambda \quad 2z=-2\lambda$

$x=\frac{\lambda}{2}, \quad y=\frac{3}{2}\lambda, \quad z=-\lambda$

$\Rightarrow \frac{\lambda}{2} + 3\left(\frac{3}{2}\lambda\right) - 2(-\lambda) = 4 \Rightarrow \frac{\lambda}{2} + \frac{9}{2}\lambda + 2\lambda = 4$

$\Rightarrow 7\lambda = 4 \Rightarrow \lambda = \frac{4}{7}$

$x=\frac{2}{7}, \ y=\frac{6}{7}, \ z=-\frac{4}{7}$

$d=\sqrt{\left(\frac{2}{7}\right)^2+\left(\frac{6}{7}\right)^2+\left(-\frac{4}{7}\right)^2} = \frac{1}{7}\sqrt{4+36+16} = \frac{\sqrt{56}}{7} = \frac{2\sqrt{14}}{7}$

Answer: $\boxed{d=\frac{2\sqrt{14}}{7}}$

3. *(10 points)* Take the region $R=\{(x,y): 0\le x\le 6, 0\le y\le 4\}$, the function $f(x,y)=x^2+2y^2$, and the partition P of R into six equal squares by the lines x = 2, x = 4, and y = 2. Approximate $\iint_R f(x,y)\,dA$ by calculating the corresponding Riemann sum $\sum_{k=1}^{6} f(\bar{x}_k,\bar{y}_k)\Delta A_k$, assuming that $(\bar{x}_k,\bar{y}_k)$ are the centers of the six squares.

$f(1,1)=3 \qquad f(1,3)=19$

$f(3,1)=11 \qquad f(3,3)=27$

$f(5,1)=27 \qquad f(5,3)=43$

$V \approx (3+11+27+19+27+43)(4) = 520$

Answer: $V \approx 520$