Math2210 Quiz 7 (Sections 12.5-12.8) Summer, 2010 Dylan Zwick

Name Solutions Date 7/26/2010

Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. Find $\frac{\partial w}{\partial t}$ for $w = x^2 y$ given $x = st$, and $y = s - t$.

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t}$$

$$\frac{\partial w}{\partial x} = 2xy \quad \frac{\partial x}{\partial t} = s \quad \frac{\partial w}{\partial y} = x^2 \quad \frac{\partial y}{\partial t} = -1$$

$$\frac{\partial w}{\partial t} = (2xy)s - x^2$$

$$= 2(st)(s-t)s - (st)^2$$

$$= 2s^3t - 2s^2t^2 - s^2t^2 = 2s^3t - 3s^2t^2$$

Answer: $\boxed{s^2t(2s-3t)}$

2. Find the equation of the tangent plane to $z = \frac{x^2}{4} + \frac{y^2}{4}$ at $(2,2,2)$.

$$\frac{\partial z}{\partial x} = \frac{x}{2} \qquad \frac{\partial z}{\partial x}(2,2) = 1$$

$$\frac{\partial z}{\partial y} = \frac{y}{2} \qquad \frac{\partial z}{\partial y}(2,2) = 1$$

$$z = z(2,2) + 1(x-2) + 1(y-2)$$

$$= 2 + x - 2 + y - 2$$

$$\Rightarrow z = x + y - 2 \Rightarrow 2 = x + y - z$$

Answer: $\boxed{2 = x + y - z}$

3. Express the number 42 as a sum of three positive numbers such that the product of these three numbers is a maximum.

$x+y+z=42$

$z=42-x-y$

$xyz = xy(42-x-y) = 42xy-x^2y-xy^2$

$= f(x,y)$

$\Rightarrow f_x(x,y) = 42y-2xy-y^2$ $\quad f_y(x,y) = 42x-2xy-x^2$

$f_x(x,y)=0 \Rightarrow 2xy = 42y-y^2 \Rightarrow x = 21-\frac{y}{2}$

$\Rightarrow f_y(x,y) = 42(21-\frac{y}{2})-2(21-\frac{y}{2})y-(21-\frac{y}{2})^2 = 0$

$\Rightarrow 882-21y-42y+y^2-441+21y-\frac{y^2}{4} = 0$

$\Rightarrow \frac{3y^2}{4}-42y+441=0$ $\quad y = \frac{42 \pm \sqrt{(-42)^2-4(\frac{3}{4})(441)}}{2(\frac{3}{4})}$

$y = \frac{42 \pm \sqrt{441}}{\frac{3}{2}} = \frac{21}{\frac{3}{2}} = 14$

~~Note: On boundary xyz~~

$x = 21-\frac{14}{2} = 14$ $\quad z = 14$

Answer: $42 = 14+14+14$

4. Find all critical points of the function $f(x,y)=x^2+4y^2-4x$.

No boundary points or singular points

$f_x = 2x-4$

$f_y = 8y$

Critical point

$2x-4=0 \Rightarrow x=2$

$8y=0 \Rightarrow y=0$

So, $(2,0)$

Check:

$f_{xx} = -2y$

$f_{yy} = -2x$

$f_{xy} = 42-2x-2y$

$= 4xy-(42-2x-2y)^2$

$= 4(14)(14)-(-14)^2$

$= 3(14)^2 > 0$,

$f_{xx} < 0$

$\therefore$ local max.

Answer : $(2,0)$.