Math2210 Quiz 6 (Sections 12.4-12.5) Summer, 2010 Dylan Zwick

Name: Solutions Date: 7/21/2010

Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. Find the directional derivative of $f(x,y)=2x^4-2x+5xy^2+xy$ at $p=(1,1)$ in the direction of $\boldsymbol{a}=2\boldsymbol{i}-3\boldsymbol{j}$.

$f_x(x,y) = 8x^3-2+5y^2+y$

$\|\vec{a}\| = \sqrt{2^2+(-3)^2} = \sqrt{13}$

$f_y(x,y) = 10xy + x$

$f_x(1,1) = 12 \quad f_y(1,1) = 11$

$\hat{a} = \dfrac{\vec{a}}{\|\vec{a}\|} = \left\langle \dfrac{2}{\sqrt{13}}, -\dfrac{3}{\sqrt{13}} \right\rangle$

$\hat{a}\cdot\nabla f(1,1) = \dfrac{24}{\sqrt{13}} - \dfrac{33}{\sqrt{13}} = -\dfrac{9}{\sqrt{13}}$

Answer: $-9/\sqrt{13}$

2. In what direction $\boldsymbol{u}$ does $f(x,y)=4+x^3y-x^2y^2$ increase most rapidly at $p=(1,2)$?

$\nabla f = \langle 3x^2y-2xy^2, x^3-2x^2y \rangle$

$\nabla f(1,2) = \langle 6-8, 1-4 \rangle = \langle -2,-3 \rangle$

The direction of $\langle -2,-3\rangle$.

A unit vector in this direction is $\left\langle -\dfrac{2}{\sqrt{13}}, -\dfrac{3}{\sqrt{13}} \right\rangle$

Any vector in this direction is fine.

Answer: $\left\langle -\dfrac{2}{\sqrt{13}}, -\dfrac{3}{\sqrt{13}} \right\rangle$

3. Find ∇F given $F(x,y,z)=2x^2z-y^4-xyz^2 \cancel{=0}$.

Typo! Should just be $F(x,y,z)=2x^2z-y^4-xyz^2$.

$$\nabla F = \langle 4xz - yz^2, -4y^3 - xz^2, 2x^2-2xyz\rangle$$

$$\frac{\partial F}{\partial x} = 4xz - yz^2 \qquad \frac{\partial F}{\partial z} = 2x^2-2xyz$$

$$\frac{\partial F}{\partial y} = -4y^3 - xz^2$$

Answer: $\langle 4xz - yz^2, -4y^3 - xz^2, 2x^2-2xyz \rangle$

4. Find the equation of the tangent plane to $f(x,y)=y^2+2x^2-6x^2y^3$ at $p_o=(2,1)$.

$$f(2,1) = 1^2+2(2^2)-6(2^2)(1^3) = 1+8-24=-15$$

$$\nabla f = \langle 4x-12xy^3, 2y-18x^2y^2\rangle$$

$$\nabla f(2,1) = \langle 8-24, 2-72\rangle = \langle -16, -70\rangle$$

$$T(x,y) = -15 + (-16)(x-2) + (-70)(y-1)$$

$$= -15 - 16x + 32 - 70y + 70$$

$$= -16x - 70y + 87$$

Tangent Plane: $\boxed{16x + 70y + z = 87}$