

Name Solutions Date 7/19/2010

Instructions: Please show all of your work as partial credit will be given where appropriate, and there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. Let $f(x, y) = \frac{3xy^3 - 2\sqrt{x^2 + 2y^4}}{x^2 + y^2}$

(a) Find $f(2, 1) = \frac{6 - 2\sqrt{6}}{5}$

$$\frac{3(2)(1^3) - 2\sqrt{(2)^2 + 2(1^4)}}{(2)^2 + (1)^2} = \frac{6 - 2\sqrt{6}}{5}$$

(b) Find $f(t, t^2) = \frac{3t^6 - 2\sqrt{1 + 2t^6}}{t + t^3}$

$$\frac{3t(t^2)^3 - 2\sqrt{t^2 + 2(t^2)^4}}{t^2 + (t^2)^2} = \frac{3t^7 - 2t\sqrt{1 + 2t^6}}{t^2 + t^4} = \frac{3t^6 - 2\sqrt{1 + 2t^6}}{t + t^3}$$

(c) What is the domain? Domain: $x^2 + y^2 \neq 0 \Rightarrow (x, y) \neq (0, 0)$

Nevermind

(d) Find $f_x(2, 1) =$

~~$$f_x(x, y) = \frac{(x^2 + y^2)(3y^3 - 2(\frac{1}{2})(x^2 + 2y^4)(2x))}{(x^2 + y^2)^2} - \frac{(3xy^3 - 2\sqrt{x^2 + 2y^4})(2x)}{(x^2 + y^2)^2}$$

$$= \frac{(x^2 + y^2)(3y^3 - 2x(x^2 + 2y^4)) - 2x(3xy^3 - 2\sqrt{x^2 + 2y^4})}{(x^2 + y^2)^2}$$~~

2. Find the slope of the tangent to the curve of intersection of the surface $z = 4x^2 + 3y - 7$ and the plane $y = 1$ at the point $(2, 1, 12)$.

$$\frac{\partial z}{\partial x} = 8x \quad \frac{\partial z}{\partial x} (2, 1, 12) = 16$$

slope = 16

3. Find the limit (or show that it does not exist).

(a) $\lim_{(x,y) \rightarrow (0,0)} \frac{\tan \sqrt{x^2 + y^2}}{4\sqrt{x^2 + y^2}}$

$$= \lim_{r \rightarrow 0} \frac{\tan r}{4r} = \frac{1}{4}$$

Answer: $\frac{1}{4}$

(b) $\lim_{(x,y) \rightarrow (0,0)} \frac{4x^2y}{x^2 + y^3}$

Along $x = 0$

$$\lim_{y \rightarrow 0} \frac{0}{y^3} = 0$$

Along $y = x$

$$\lim_{x \rightarrow 0} \frac{4x^3}{2x^3} = 2$$

Answer: Does not exist.