

Name Solutions Date 7/18/2010

Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. Name the type of quadric surface given by $4x^2 + 4y^2 - 25z^2 + 100 = 0$

$$4x^2 + 4y^2 - 25z^2 = -100$$
$$\Rightarrow \frac{z^2}{4} - \frac{x^2}{25} - \frac{y^2}{25} = 1$$

Form of hyperboloid of two sheets.

Type of surface: Hyperboloid of two sheets.

2. Name the type of quadric surface given by $4x^2 + 9y^2 + 49z^2 = 1764$

$$\frac{x^2}{441} + \frac{y^2}{196} + \frac{z^2}{36} = 1$$
$$= \left(\frac{x}{21}\right)^2 + \left(\frac{y}{14}\right)^2 + \left(\frac{z}{6}\right)^2 = 1$$

Type of surface: Ellipsoid

3. a) Change $(6, \frac{\pi}{6}, -2)$ from cylindrical coordinates to Cartesian.

$$x = r \cos \theta = 6 \cos\left(\frac{\pi}{6}\right) = 6\left(\frac{\sqrt{3}}{2}\right) = 3\sqrt{3}$$

$$y = r \sin \theta = 6 \sin\left(\frac{\pi}{6}\right) = 6\left(\frac{1}{2}\right) = 3$$

Answer: $(3\sqrt{3}, 3, -2)$

- b) Change $(2, 2, 3)$ from Cartesian coordinates to cylindrical.

$$r = \sqrt{2^2 + 2^2} = 2\sqrt{2}$$

$$\theta = \tan^{-1}\left(\frac{1}{1}\right) = \pi/4$$

Answer: $(2\sqrt{2}, \pi/4, 3)$

4. Change $x^2 + y^2 + 4z^2 = 10$ to the following coordinates:

- a) Spherical

$$\begin{aligned} x &= \rho \cos \theta \sin \phi & \rho^2 \cos^2 \theta \sin^2 \phi + \rho^2 \sin^2 \theta \sin^2 \phi + 4\rho^2 \cos^2 \phi &= 10 \\ y &= \rho \sin \theta \sin \phi & \Rightarrow \rho^2 \sin^2 \phi + \rho^2 \cos^2 \phi + 3\rho^2 \cos^2 \phi &= 10 \\ z &= \rho \cos \phi & \Rightarrow \rho^2 (1 + 3 \cos^2 \phi) &= 10 \end{aligned}$$

Answer: $\rho^2 (1 + 3 \cos^2 \phi) = 10$

- b) Cylindrical

$$\begin{aligned} x &= r \cos \theta & x^2 + y^2 + 4z^2 &= 10 \\ y &= r \sin \theta & r^2 \cos^2 \theta + r^2 \sin^2 \theta + 4z^2 &= 10 \\ & & r^2 + 4z^2 &= 10 \end{aligned}$$

Answer: $r^2 + 4z^2 = 10$