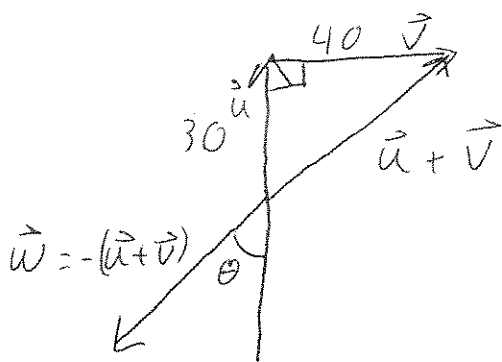


Name Solutions Date 7/14/10

Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. Force \mathbf{u} has a magnitude of 30 pounds in the North direction. Force \mathbf{v} has a magnitude of 40 pounds in the East direction. Find the magnitude and direction (geometrically) of the force \mathbf{w} needed to counterbalance \mathbf{u} and \mathbf{v} . (Just write answers in as simplified a form as you can without a calculator.)



$$|\vec{u} + \vec{v}| = \sqrt{30^2 + 40^2} = 50$$

$$\vec{w} = -(\vec{u} + \vec{v})$$

$$\cos \theta = \frac{40}{50} \Rightarrow \theta = \cos^{-1}\left(\frac{4}{5}\right)$$

So, $\cos^{-1}\left(\frac{4}{5}\right)$ West of South.

magnitude of \mathbf{w} : 50 pounds

direction of \mathbf{w} : $\cos^{-1}\left(\frac{4}{5}\right)$ West of South.

2. For $\mathbf{u} = \langle -2, 5, 1 \rangle$ and $\mathbf{v} = 3\mathbf{i} + 1\mathbf{j} - 5\mathbf{k}$,

- (a) find $\mathbf{u} + 2\mathbf{v}$.

$$\begin{aligned} \vec{u} + 2\vec{v} &= \langle -2 + 2(3), 5 + 2(1), 1 + 2(-5) \rangle \\ &= \langle 4, 7, -9 \rangle \end{aligned}$$

$$\mathbf{u} + 2\mathbf{v} = \underline{\langle 4, 7, -9 \rangle}$$

- (b) find $\hat{\mathbf{u}}$.

$$\|\vec{u}\| = \sqrt{(-2)^2 + 5^2 + 1^2} = \sqrt{30}$$

$$\hat{\mathbf{u}} = \frac{1}{\sqrt{30}} \vec{u} = \frac{1}{\sqrt{30}} \langle -2, 5, 1 \rangle$$

$$\hat{\mathbf{u}} = \underline{\left\langle -\frac{2}{\sqrt{30}}, \frac{5}{\sqrt{30}}, \frac{1}{\sqrt{30}} \right\rangle}$$

3. Find the projection of $\langle 2, 1, -1 \rangle$ onto the vector $\langle 1, 5, 3 \rangle$

$$\text{proj}_{\langle 1, 5, 3 \rangle} \langle 2, 1, -1 \rangle = \frac{\langle 2, 1, -1 \rangle \cdot \langle 1, 5, 3 \rangle}{\langle 1, 5, 3 \rangle \cdot \langle 1, 5, 3 \rangle} \langle 1, 5, 3 \rangle$$

$$= \frac{2 + 5 - 3}{1^2 + 5^2 + 3^2} \langle 1, 5, 3 \rangle$$

$$= \frac{4}{35} \langle 1, 5, 3 \rangle = \left\langle \frac{4}{35}, \frac{4}{7}, \frac{12}{35} \right\rangle$$

projection: $\frac{4}{35} \langle 1, 5, 3 \rangle$

4. Circle all of the following statements that make sense.

(a) $u \cdot (v + w)$

(b) $|u|(v + w)$

(c) $(u \cdot v)|w|$

(d) $(u \cdot v) \cdot w$