Math2210 Quiz 12 (Sections 14.2-14.3) Summer, 2010 Dylan Zwick

Name Solutions Date 8/3/2010

Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. (15 points) Determine whether $\boldsymbol{F}(x,y)=(2e^{y}-ye^{x})\boldsymbol{i}+(2xe^{y}-e^{x})\boldsymbol{j}$ is conservative. If so, find f such that $\boldsymbol{F}=\nabla f$. If not, state that $\boldsymbol{F}$ is not conservative.

$M = 2e^y - ye^x \quad N = 2xe^y - e^x$

$\frac{\partial N}{\partial x} = 2e^y - e^x \quad \frac{\partial M}{\partial y} = 2e^y - e^x$ ✓

So, conservative.

$\frac{\partial f}{\partial x} = 2e^y - ye^x \Rightarrow f = 2xe^y - ye^x + g(y)$

$\frac{\partial f}{\partial y} = 2xe^y - e^x + g'(y) = 2xe^y - e^x \Rightarrow g'(y) = 0$

$\Rightarrow g(y) = C$

$\Rightarrow \boxed{f(x,y) = 2xe^y - ye^x + C}$

Conservative (True) or False (circle one)

If conservative, f = $2xe^y - ye^x + C$

2. *(15 points)* Evaluate the line integral $\int_C xe^y \, ds$, where C is the line segment from (-1,2) to (1,1).

$x = -1 + 2t \qquad t \in [0,1] \qquad dx = 2dt$

$y = 2 - t \qquad\qquad\qquad dy = -dt$

$\Rightarrow \quad ds = \sqrt{(dx)^2 + (dy)^2}$

$= \sqrt{2^2 + (-1)^2}\, dt = \sqrt{5}\, dt$

$$\int_0^1 (-1+2t)\, e^{2-t} \sqrt{5}\, dt$$

$$= \sqrt{5} e^2 \int_0^1 (2t-1) e^{-t} dt$$

$$= \sqrt{5} e^2 \left[2\int_0^1 t e^{-t} dt - \int_0^1 e^{-t} dt \right]$$

$u = t \quad dv = e^{-t} dt$

$du = dt \quad v = -e^{-t}$

$$= \sqrt{5} e^2 \left[2\left[-te^{-t} \Big|_0^1 + \int_0^1 e^{-t} dt \right] - \int_0^1 e^{-t} dt \right]$$

$$= \sqrt{5} e^2 \left[2(-e^{-1}) + \int_0^1 e^{-t} dt \right]$$

$$= \sqrt{5} e^2 (-2e^{-1} - e^{-1} + 1)$$

$$= \sqrt{5} e^2 (1 - 3e^{-1})$$

Answer: $\boxed{\sqrt{5}(e^2 - 3e)}$

3. *(10 points)* What.... is your favorite color?

(Answers may vary.)

Answer: Yellow.