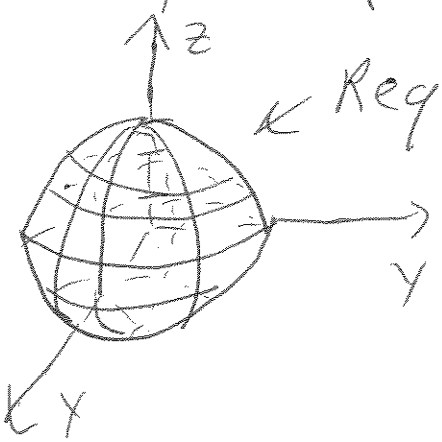
Math2210 Quiz 11 (Sections 13.8, 14.1) Summer, 2010 Dylan Zwick

Name Solutions Date 8/2/2010

Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. (14 points) Evaluate $\int_{-3}^{3}\int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}}\int_{-\sqrt{9-x^2-z^2}}^{\sqrt{9-x^2-z^2}} (x^2+y^2+z^2)^{3/2}\,dy\,dz\,dx$.

$x^2+y^2+z^2=9$



Convert to spherical!

$$\int_0^{\pi}\int_0^{2\pi}\int_0^{3} \rho^5 \sin\phi \, d\rho\, d\theta\, d\phi$$

$$= \int_0^{\pi}\int_0^{2\pi} \frac{\rho^6}{6}\Big|_0^3 \sin\phi\, d\theta\, d\phi$$

$$= \frac{243}{2}\int_0^{\pi}\int_0^{2\pi} \sin\phi\, d\theta\, d\phi$$

$$= 243\pi \int_0^{\pi} \sin\phi\, d\phi$$

$$= 243\pi - \cos\phi\Big|_0^{\pi}$$

$$= 243\pi\left(-(-1)-(-1)\right)$$

$$= 486\pi$$

Answer : $\boxed{486\pi}$

2. *(14 points)* Calculate the determinant of the Jacobian J(u,v) for the change of variables $x = u^2 - 2uv$ $\quad y = v^3 + 3uv^2$

$$\frac{\partial x}{\partial u} = 2u - 2v \qquad \frac{\partial x}{\partial v} = -2u$$

$$\frac{\partial y}{\partial u} = 3v^2 \qquad \frac{\partial y}{\partial v} = 3v^2 + 6uv$$

$$J(u,v) = \begin{pmatrix} 2u-2v & -2u \\ 3v^2 & 3v^2+6uv \end{pmatrix}$$

$$\Rightarrow |J(u,v)| = (2u-2v)(3v^2+6uv) - (-2u)(3v^2)$$

$$= 6uv^2 + 12u^2v - 6v^3 - 12uv^2 + 6uv^2$$

$$= 12u^2v - 6v^3 = \boxed{6v(2u^2 - v^2)}$$

Absolute value: $6\,|v(2u^2 - v^2)|$

3. *(3 points each)* If f(x,y,z) is a scalar function and **F**(x,y,z) is a vector field, which of the following make sense (circle one):

a) $\nabla \cdot \nabla(f)$ Makes sense. Does not make sense.

b) div(grad(f)) Makes sense. Does not make sense.

c) grad(div(**F**)) Makes sense. Does not make sense.

d) $\nabla \times \nabla(f)$ Makes sense. Does not make sense.