Math2210 Quiz 10 (Sections 13.6, 13.7) Summer, 2010 Dylan Zwick

Name: Solutions Date: 7/29/2010

Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. *(10 points) Find the area of the plane* $3x+2y+6z=12$ *that is above the rectangle in the xy-plane with vertices (0,0), (2,0), (2,1), and (0,1).*

$$z = 2 - \frac{1}{2}x - \frac{1}{3}y \qquad \frac{\partial z}{\partial x} = -\frac{1}{2} \qquad \frac{\partial z}{\partial y} = -\frac{1}{3}$$

$$SA = \int_0^2\int_0^1 \sqrt{1+\left(-\tfrac{1}{2}\right)^2+\left(-\tfrac{1}{3}\right)^2}\,dy\,dx = \int_0^2\int_0^1 \sqrt{\frac{36}{36}+\frac{9}{36}+\frac{4}{36}}\,dy\,dx$$

$$= \frac{7}{6}\int_0^2\int_0^1 dy\,dx = \frac{7}{3}$$

Answer: $\boxed{\frac{7}{3}}$

2. *(15 points) Evaluate the iterated integral* $\int_0^2\int_1^z\int_0^{\sqrt{x/z}} 2xyz\,dy\,dx\,dz$.

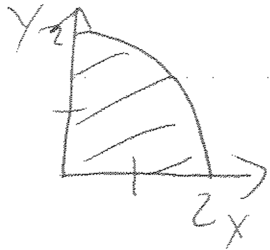
$$\int_0^2\int_1^z xy^2z\Big|_0^{\sqrt{\frac{x}{z}}}\,dx\,dz = \int_0^2\int_1^z x^2\,dx\,dz$$

$$= \int_0^2 \frac{x^3}{3}\Big|_1^z\,dz = \int_0^2 \frac{z^3}{3}-\frac{1}{3}\,dz$$

$$= \frac{z^4}{12}-\frac{z}{3}\Big|_0^2 = \frac{16}{12}-\frac{2}{3} = \frac{4}{3}-\frac{2}{3} = \frac{2}{3}$$

Answer: $\boxed{\frac{2}{3}}$

3. *(15 points)* Find the area of the surface $z=\sqrt{4-y^2}$ in the first octant that is directly above the circle $x^2+y^2=4$ in the xy-plane.



$$\frac{\partial z}{\partial x} = 0 \qquad \frac{\partial z}{\partial y} = \frac{-y}{\sqrt{4-y^2}}$$

$$\sqrt{1+0^2+\left(\frac{-y}{\sqrt{4-y^2}}\right)^2} = \sqrt{\frac{4}{4-y^2}} = \frac{2}{\sqrt{4-y^2}}$$

$$\int_0^2 \int_0^{\sqrt{4-x^2}} \frac{2}{\sqrt{4-y^2}}\, dy\, dx$$ Convert to polar

$u = 4 - r^2\sin^2\theta$

$du = -2r\sin^2\theta\, dr$

$-\dfrac{du}{2\sin^2\theta} = r\,dr$

$$2\int_0^{\pi/2}\int_0^2 \frac{r}{\sqrt{4-r^2\sin^2\theta}}\, dr\, d\theta$$

$$= 2\int_0^{\pi/2}\int_4^{4(1-\sin^2\theta)} -\frac{du}{2\sin^2\theta\sqrt{u}}$$

$$= 2\int_0^{\pi/2}\int_{4\cos^2\theta}^{4} \frac{du}{2\sin^2\theta\sqrt{u}} = 2\int_0^{\pi/2} \frac{\sqrt{u}}{\sin^2\theta}\Bigg|_{4\cos^2\theta}^{4}$$

$$= 2\int_0^{\pi/2}\left(\frac{2}{\sin^2\theta}\right) - \frac{2\cos\theta}{\sin^2\theta} = 4\int_0^{\pi/2}\left(\frac{1-\cos\theta}{\sin^2\theta}\right) d\theta$$

$$= 4\int_0^{\pi/2}\left(\frac{1}{1+\cos\theta}\right) d\theta = 4\tan\left(\frac{\theta}{2}\right)\Bigg|_0^{\pi/2}$$

$$= 4\tan\left(\frac{\pi}{4}\right) - 0 = 4$$

Answer: $\boxed{4}$