

Name _____ Date _____

Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. (15 points each) Evaluate the integrals.

(a) $\int_0^2 \int_1^3 x^2 y \, dy \, dx$

Answer 1(a): _____

(b) $\int_1^5 \int_0^x \frac{3}{x^2 + y^2} \, dy \, dx$

Answer 1(b): _____

(Note: This is #1 continued!)

$$(c) \int_0^{\pi/2} \int_0^z \int_0^y \sin(x+y+z) \, dx \, dy \, dz$$

Answer 1(c): _____

2. (15 points) Find the area of the smaller region bounded by $\theta = \pi/6$ and $r = 4 \sin \theta$.

Answer 2: _____

3. (15 points) If $R = \{(x, y) : 0 \leq x \leq 6, 0 \leq y \leq 4\}$ and P is the partition of R into six equal squares by the lines $x = 2$, $x = 4$, and $y = 2$. Approximate $\iint_R f(x, y) dA$ by calculating the corresponding Riemann sum $\sum_{k=1}^6 f(\bar{x}_k, \bar{y}_k) \Delta A_k$, assuming that (\bar{x}_k, \bar{y}_k) are the centers of the six squares. Take $f(x, y) = 12 - x - y$.

Answer 3: _____

4. (20 points) Find the volume of the solid in the first octant bounded by the surface $z = 9 - x^2 - y^2$ and the coordinate planes.

Answer 4: _____

5. For the integral $V = \int_0^2 \int_0^{4-2y} \int_0^{4-2y-z} dx \, dz \, dy$, do the following.

- (a) (15 points) Rewrite it, changing the order of integration to $dz \, dy \, dx$.
(Don't evaluate it, just set it up.)

Answer 5(a): _____

(b) (15 points) For the integral $V = \int_0^{\sqrt{65}} \int_4^{\sqrt{81-y^2}} \int_0^{\sqrt{81-y^2-z^2}} dx \, dz \, dy$, rewrite it using spherical coordinates. (Don't evaluate it, just set it up.)

Answer 5(b): _____

6. (20 points) Find the surface area of the part of the paraboloid $z = x^2 + y^2$ that is cut off by the plane $z = 4$.

Answer 6: _____

7. (25 points) Find the maximum volume of a closed rectangular box with faces parallel to the coordinate planes inscribed in the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \quad .$$

Answer 7: _____

8. Find the divergence and curl of the following vector fields. (15 points each)

a) $\mathbf{F}(x, y, z) = yz \mathbf{i} + xz \mathbf{j} + xy \mathbf{k}$

Divergence 8a): _____

Curl 8a): _____

b) $\mathbf{F}(x, y, z) = x^2 \mathbf{i} - 2xy \mathbf{j} + yz^2 \mathbf{k}$

Divergence 8b): _____

Curl 8b): _____

Extra Credit (15 points): Calculate $\int_{-\infty}^{\infty} e^{-x^2} dx$. Note that you must show your calculation, you cannot just state the answer.

Extra Credit Answer: _____