Math2210 Midterm 1 Summer, 2010 Dylan Zwick

Name _____ Date _____

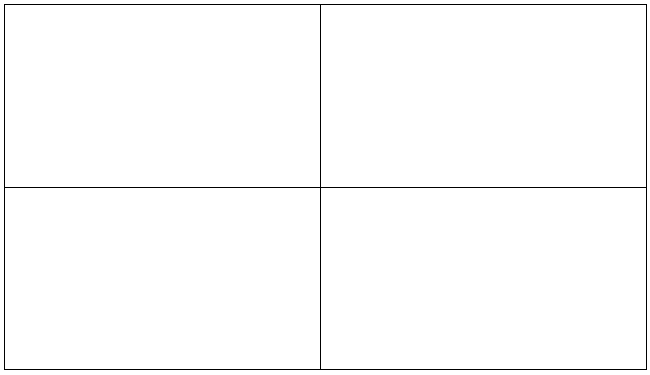
Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. For $x = t^3 - 4t$ and $y = t^2 - 4$ such that $-3 \leq t \leq 3$, do the following:

   (a) (10 pts) Eliminate the parameter to obtain the corresponding Cartesian equation.

Answer 1(a): _____

(b) (10 pts) Graph the curve.



(c) (5 pts) Indicate if the curve is simple and/or closed.

Simple: T or F (circle one)

Closed: T or F (circle one)

2. (10 pts) Find the length of the curve given by $x = t + \frac{1}{t}$ and $y = \ln t^2$ for $1 \le t \le 4$.

Answer 2: _____

3. (15 pts) For position vector given by $\boldsymbol{r}(t) = t^3 \boldsymbol{i} + t^2 + 2t \boldsymbol{j} + \ln t \boldsymbol{k}$, find the velocity and acceleration vectors and the speed at $t = 3$.

$\boldsymbol{v}(t)$ = _____

$\boldsymbol{a}(t)$ = _____

speed at $t = 3$ = _____

4. (10 pts) Find the limit, if it exists. $\lim_{t \to 0} \left[\frac{3t^2 \tan t}{2t^3} \boldsymbol{i} - \frac{4t}{t^2 - 1} \boldsymbol{j} + \frac{3t^2}{1 - \cos^2 t} \boldsymbol{k} \right]$

Answer (4) : _____

5. (10 pts) Find the equation of the sphere that has the line segment joining (0, 2, 3) and (4, 0, 5) as a diameter.

Radius = _____ units

center = _____

Eqn of sphere: _____

6. (10 pts each) Let $\boldsymbol{a}=\langle 2,0,3\rangle$, $\boldsymbol{b}=\langle -3,1,4\rangle$ and $\boldsymbol{c}=5\boldsymbol{i}+2\boldsymbol{k}$. Find each of the following.

(a) $2\boldsymbol{a}-3\boldsymbol{c}$

$2\boldsymbol{a}-3\boldsymbol{c}$ = _____

(b) $\boldsymbol{a}\cdot(\boldsymbol{b}+\boldsymbol{c})$

$\boldsymbol{a}\cdot(\boldsymbol{b}+\boldsymbol{c})$ = _____

(c) $\boldsymbol{b}\cdot\boldsymbol{c}-|\boldsymbol{b}|$

$\boldsymbol{b}\cdot\boldsymbol{c}-|\boldsymbol{b}|$ = _____

(Note: This is # 6 continued.) $\boldsymbol{a}=\langle 2,0,3\rangle$, $\boldsymbol{b}=\langle -3,1,4\rangle$ and $\boldsymbol{c}=5\boldsymbol{i}+2\boldsymbol{k}$

(d) $\hat{\boldsymbol{c}}$ (the unit vector)

$\hat{\boldsymbol{c}}$ = _____

(e) $\boldsymbol{a}\times(\boldsymbol{b}\times\boldsymbol{c})$

$\boldsymbol{a}\times(\boldsymbol{b}\times\boldsymbol{c})$ = _____

(f) $\boldsymbol{a}\cdot(\boldsymbol{b}\times\boldsymbol{c})$

$\boldsymbol{a}\cdot(\boldsymbol{b}\times\boldsymbol{c})$ = _____

7. (10 pts each) For $\boldsymbol{a}=2\boldsymbol{i}-\boldsymbol{j}+3\boldsymbol{k}$ and $\boldsymbol{b}=\boldsymbol{i}+2\boldsymbol{j}-\boldsymbol{k}$, find each of the following:

(a) Direction cosines for $\boldsymbol{a}$.

$\cos\alpha$ = _____

$\cos\beta$ = _____

$\cos\gamma$ = _____

(b) The angle θ between $\boldsymbol{a}$ and $\boldsymbol{b}$. (Just write a simplified expression. If you don't have a calculator just write the numerical formula for the angle.)

θ = _____

(c) Find the projection of $\boldsymbol{b}$ onto $\boldsymbol{a}$.

Projection of $\boldsymbol{b}$ onto $\boldsymbol{a}$ = _____

8. (10 pts each) For the planes given by $4x - y + 2z = 7$ and $5x + 3z = 13$, answer the following questions.

(a) Find the line of intersection between the planes and write that line in parametric equations.

Line: _____

(b) Find the equation of the plane that is perpendicular to the line of intersection and goes through the point (0, 2, 1).

Equation of plane: _____

9. (a) (10 pts) Convert $2x^2 + 2y^2 = 5y + 81$ from a Cartesian coordinate equation into an equation in cylindrical coordinates.

Answer : _____

(b) (10 pts) Convert $\rho = -3\sec\phi$ from a spherical coordinate equation into an equation in Cartesian coordinates.

Answer : _____

Extra Credit: (10 pts)

A luxury cruiseliner is traveling due west at only 8 miles per hour. A woman on the ship is running across the ship, heading due north, at 6 miles per hour. What are the magnitude and direction of her velocity relative to the surface of the water? (If you don't have a calculator, just give the angle in simplified form.)

velocity magnitude: _____

velocity direction: _____