Math2210 Final Exam Summer, 2010 Dylan Zwick

Name _____ Date _____

Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. *(15 points)* For position vector given by $\boldsymbol{r}(t)=(t^4-3t^2-t)\boldsymbol{i}+(t^3+t)\boldsymbol{j}$, find the velocity and acceleration vectors and the speed at $t=1$.

$\boldsymbol{v}(t)$ = _____

$\boldsymbol{a}(t)$ = _____

speed at $t=1$ = _____

2. *(20 points)* Let $\boldsymbol{a}=\langle 1,-3,2\rangle$, $\boldsymbol{b}=\langle 2,6,3\rangle$ and $\boldsymbol{c}=\langle -2,5,0\rangle$. Find each of the following.

(a) $2\boldsymbol{a}-3\boldsymbol{c}$

$2\boldsymbol{a}-3\boldsymbol{c}$ = _____

(b) $\boldsymbol{a}\cdot(\boldsymbol{b}+\boldsymbol{c})$

$\boldsymbol{a}\cdot(\boldsymbol{b}+\boldsymbol{c})$ = _____

(c) projection of $\boldsymbol{a}$ onto $\boldsymbol{b}$

projection of $\boldsymbol{a}$ onto $\boldsymbol{b}$ = _____

(d) $\hat{\boldsymbol{a}}$ (the unit vector)

$\hat{\boldsymbol{a}}$ = _____

3. For the points A(-2, 4, 3), B(4, 2, 3) and C(1, 2, -1)

(a) *(10 points)* Write the equation of the plane through points A, B and C.

plane normal vector = _____

Equation of plane: _____

(b) *(10 points)* Write a set of parametric equations for the line through point B and perpendicular to the plane in part (a).

Line: _____

4. *(15 points)* Find the directional derivative of $f(x, y, z) = x^3 y - y^2 z^2 + x^2 y^2 z^2$ at $\boldsymbol{p} = (1, 1, 1)$ in the direction of $\boldsymbol{a} = \boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$.

Answer 4: _____

5. *(20 points)* For the surface $F(x, y, z) = 4x^2 + 2xy + 2y^2 - 4yz - z^2 + x + z = 5$

(a) Find the equation of the tangent plane at the point (1, 1, 1).

Answer 5(a): _____

(b) Find a point on the surface where the tangent plane is parallel to the plane $9x - 2y - z = 42$.

Answer 5(b):_____

6. *(15 points)* Find all critical points of the function $f(x, y) = xy + \frac{2}{x} + \frac{2}{y}$.

Determine if each critical point is a local minimum, a local max, or neither. If there are none of the given type of point, just write "None".

Critical Points: _____

Local Max:_____
Local Min:_____
Neither:_____

7. (20 points) Find the minimum distance between the origin and the plane $x+3y-2z=4$.

Answer:_____

8. *(20 points)* Given $\boldsymbol{F}(x, y, z) = 5x^3 y z\,\boldsymbol{i} - 2y x^2\,\boldsymbol{j} + y^3 z^2\,\boldsymbol{k}$, calculate the following.

(a) div $\boldsymbol{F}$

div $\boldsymbol{F}$ = _____

(b) curl $\boldsymbol{F}$

curl $\boldsymbol{F}$ = _____

(c) $\nabla(\nabla \cdot \boldsymbol{F})$

$\nabla(\nabla \cdot \boldsymbol{F})$ = _____

(d) $\nabla \cdot (\nabla \times \boldsymbol{F})$

$\nabla \cdot (\nabla \times \boldsymbol{F})$ = _____

9. *(15 points)* Evaluate the line integral $\int_C (x^2+y^2)\,ds$ given C is the path given by $x=e^t \sin t$, $y=e^t \cos t$ and $0 \le t \le 3$.

Answer 9: _____

10. *(20 points)* Determine whether

$$\boldsymbol{F}(x, y, z) = (3x^2y^2z^2 + z^2 + y)\boldsymbol{i} + (2x^3yz^2 + 2yz + x + 1)\boldsymbol{j} + (2x^3y^2z + y^2 + 2xz)\boldsymbol{k}$$

is conservative. If so, find f such that $\boldsymbol{F} = \nabla f$. If not, state that $\boldsymbol{F}$ is not conservative.

Conservative: True or False (circle one)

If conservative, f = _____

11. *(20 points)* Evaluate $\oint_C (x^2+4xy)\,dx+(2x^2+3y)\,dy$ where C is the ellipse $9x^2+16y^2=144$ oriented counter-clockwise.

Answer 11: _____

Extra Credit:

(5 points) What is 4+9? Hint - It's not 23.

Answer : _____