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Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. (5 points) Find the length of the curve given by $x=2\cos t$ and $y=2\sin t-1$ for $0 \leq t \leq \frac{\pi}{2}$.

$$\frac{dx}{dt} = -2\sin t \quad \frac{dy}{dt} = 2\cos t$$

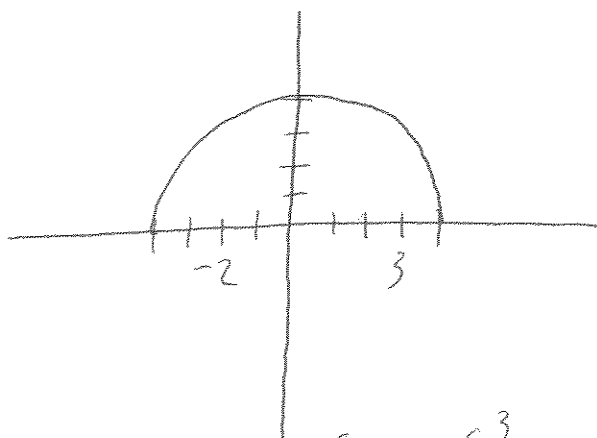
$$L = \int_0^{\pi/2} \sqrt{(-2\sin t)^2 + (2\cos t)^2} dt$$

$$= \int_0^{\pi/2} \sqrt{4\sin^2 t + 4\cos^2 t} dt = 2\sqrt{2}$$

$$= 2 \int_0^{\pi/2} \sqrt{\sin^2 t + \cos^2 t} dt = 2 \int_0^{\pi/2} 1 dt = 2\left(\frac{\pi}{2}\right) = \pi$$

Answer 1: π

2. (5 points) Find the surface area of the surface generated by revolving the curve given by $y=\sqrt{16-x^2}$ for $-2 \leq x \leq 3$ about the x-axis.



$$\frac{dy}{dx} = \frac{-x}{\sqrt{16-x^2}}$$

$$\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \sqrt{1 + \frac{x^2}{16-x^2}}$$

$$= \sqrt{\frac{16}{16-x^2}} = \frac{4}{\sqrt{16-x^2}}$$

$$SA = \int_{-2}^3 2\pi \sqrt{16-x^2} \left(\frac{4}{\sqrt{16-x^2}}\right) dx = 8\pi \int_{-2}^3 1 dx = 40\pi$$

Answer 2: 40π Note: #3 on the back side!

3. (5 points) A force of 9 pounds is required to keep a spring stretched $\frac{1}{2}$ foot beyond its normal length. Find the work done in stretching the spring 2 feet beyond its natural length.

$$9 \text{ lbs} = k \left(\frac{1}{2} \text{ ft} \right) \Rightarrow k = 18 \text{ lbs/ft}$$

$$W = \int_0^2 18x \, dx$$

$$= 18 \left(\frac{x^2}{2} \right) \Big|_0^2 = \boxed{36 \text{ ft} \cdot \text{lbs}}$$

Answer 3: _____

$$\boxed{36 \text{ ft} \cdot \text{lbs}}$$