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Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

Potentially Useful Formulas

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^n i^3 = \frac{n^2(n+1)^2}{4}$$

1. (7 pts) Solve this differential equation.

$$\frac{dy}{dx} = \frac{x+3x^2}{y^2} \quad \text{and} \quad y=2 \quad \text{when} \quad x=0$$

$$\int y^2 dy = \int (x+3x^2) dx$$

$$\frac{y^3}{3} = \frac{x^2}{2} + x^3 + C$$

$$\Rightarrow y^3 = \frac{3}{2}x^2 + 3x^3 + C$$

$$\Rightarrow y = \left(\frac{3}{2}x^2 + 3x^3 + C \right)^{1/3} \quad x=0, y=2$$

$$\Rightarrow 2 = C^{1/3} \Rightarrow 2^3 = C = 8$$

$$\text{So, } y = \left(\frac{3}{2}x^2 + 3x^3 + 8 \right)^{1/3}$$

Answer 1: $y = \left(\frac{3}{2}x^2 + 3x^3 + 8 \right)^{1/3}$

(Note: #2 and #3 are on the back!)

2. (4 points) Evaluate.

$$\begin{aligned} & \sum_{i=1}^{10} [(3i-4)(i+5)] \\ &= \sum_{i=1}^{10} (3i^2 + 11i - 20) \\ &= 3 \left(\frac{10(10+1)(2(10)+1)}{6} \right) + 11 \left(\frac{10(10+1)}{2} \right) - 20(10) \\ &= 5(11)(21) + 5(121) - 200 \\ &= 1155 + 605 - 200 = 1,560 \end{aligned}$$

Answer 2: 1,560

3. (4 points) Evaluate.

$$\int (5x^3 \sqrt{2x^4+3}) dx$$

$$g(x) = 2x^4 + 3$$

$$g'(x) = 8x^3$$

$$\begin{aligned} \int (5x^3 \sqrt{2x^4+3}) dx &= \frac{5}{8} \int g'(x) \sqrt{g(x)} dx \\ &= \frac{5}{8} \frac{[g(x)]^{3/2}}{(\frac{3}{2})} + C = \frac{5}{12} (2x^4+3)^{3/2} + C \end{aligned}$$

Answer 3: $\frac{5}{12} (2x^4+3)^{3/2} + C$