Math1210 Quiz 10 (3.2, 3.3, 3.4) Summer, 2009 Dylan Zwick

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Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. [8 points] For $f(x)=\frac{1}{2}x^2-\frac{1}{x}$

(a) For what **x** values is the function increasing? For what **x** values if the function decreasing?. [3 points]

$f'(x) = x + \frac{1}{x^2}$ $\quad f'(x) < 0$ for $x < -1$

$f'(x) = 0$ when $x = -1$ $\quad f'(x) > 0$ for $-1 < x < 0$, $x > 0$

$f''(x) = 1 - \frac{2}{x^3}$ $\quad f''(x) > 0$ for $x < 0$ and $x > \sqrt[3]{2}$

$f''(x) = 0$ when $x = \sqrt[3]{2}$ $\quad f''(x) < 0$ for $0 < x < \sqrt[3]{2}$

Increasing: $(-1, 0)$ and $(0, \infty)$

Decreasing: $(-\infty, -1]$

(b) Find all local min and max **point(s)**. (Both x and y values) [2 points]

$f'(x) = 0$ when $x = -1$

$f''(x) = 3$ when $x = -1$

Max point(s): None

Min point(s): $(-1, \frac{3}{2})$

(c) For what **x** values is the function concave up? For what **x** values is the function concave down? [2 points]

Concave up: $(-\infty, 0)$, $[\sqrt[3]{2}, \infty)$

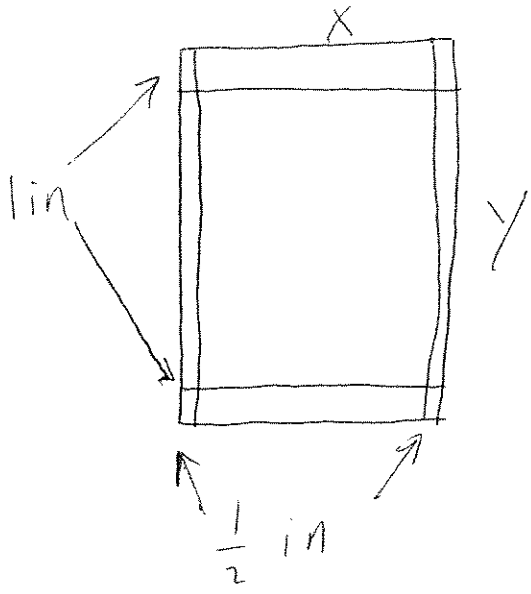
Concave down: $(0, \sqrt[3]{2}]$

(d) Find all x-values of inflection point(s). [1 point]

x-value(s) of inflection point(s): $x = \sqrt[3]{2}$

Note: As $f(x)$ is undefined at $x=0$, $x=0$ is not an inflection point. However, if you put $x=0, \sqrt[3]{2}$ I gave you 1 point.

2. [7 points] The page of a book is to have a total area of 100 square inches, with 1-inch margins at the bottom and top and ½-inch margin at the sides. Find the dimensions of the page which will allow the largest printed area.



$P = (x-1)(y-2)$

$xy = 100$

$\Rightarrow y = \frac{100}{x}$

$P(x) = (x-1)\left(\frac{100}{x} - 2\right)$

$= 100 - 2x - \frac{100}{x} + 2$

$P(x) = 102 - 2x - \frac{100}{x}$

$P'(x) = \frac{100}{x^2} - 2$, $P''(x) = -\frac{200}{x^3}$

$P'(x) = 0$ when $x = \sqrt{50} = 5\sqrt{2}$

$P''(\sqrt{50}) = -\frac{200}{50^{3/2}} < 0$

$P''(x) < 0$ for all $x > 0$.

So, $x = 5\sqrt{2}$ is a max.

$y = \frac{100}{5\sqrt{2}} = 10\sqrt{2}$

Answer 2: width $= 5\sqrt{2}$ in height $= 10\sqrt{2}$ in