Solutions for Introduction to Polynomial Calculus

Section 1 Problems

Bob Palais

The point-slope form of the equation of a line says that the rise over the run between an arbitrary point on a line (x, y) and a particular point (x_0, y_0) on that line is constant, m, called the slope of the line. This describes a relationship of direct proportionality or linearity between the rise and the run. The rise is the change in y, $y-y_0$, and the run is the change in x, $x-x_0$, so $\frac{y-y_0}{x-x_0} = m$. Since the ratio is undefined for the point (x_0, y_0), it is common to cross multiply so that this point fits the equation explicitly: $y-y_0 = m(x-x_0)$. If you are given two points on a line, they may be used to compute its slope, and either may be used in the point-slope form.

So for (1)-(6) I'm giving not only the slope which the problem asks for but also the point-slope equation of the line.

(1) $m = \frac{2-1}{1-0} = 1$ and the equation is $y-1 = 1(x-0)$ or $y-2 = 1(x-1)$.

(2) $m = \frac{7-3}{4-2} = 2$ and the equation is $y-3 = 2(x-2)$ or $y-7 = 2(x-4)$.

(3) $m = \frac{2-1}{3-1} = \frac{1}{2}$ and the equation is $y-1 = \frac{1}{2}(x-1)$ or $y-2 = \frac{1}{2}(x-3)$.

(4) $m = \frac{2-4}{3-1} = -1$ and the equation is $y-4 = -1(x-1)$ or $y-2 = -1(x-3)$.

(5) $m = \frac{1-3}{3-(-2)} = -\frac{2}{5}$ and the equation is $y-3 = -\frac{2}{5}(x-(-2))$ or $y-1 = -\frac{2}{5}(x-3)$.

(6) $m = \frac{2-0}{0-(-2)} = 1$ and the equation is $y-0 = 1(x-(-2))$ or $y-2 = 1(x-0)$.

(7) $y-0 = 2(x-0)$

(8) $y-2 = 5(x-1)$

(9) $y-(-1) = -3(x-2)$

(10) $y-1 = \frac{1}{2}(x-1)$

(11) $y-5 = -\frac{2}{3}(x-0)$

(12) $y-0 = 7(x-(-2))$

I intentionally prefer the $(x-(-a))$ form to $(x+a)$ because it displays the important information more clearly. I do not require or encourage oversimplification of answers! Conversion to slope-intercept form is not required or encouraged either as long as you know how to do it. Usually points other than $x = 0$ are more important and it is better to refer equations to the point of interest. The slope-intercept form is nice when you wish to extend to polynomials in standard form: $a_0 + a_1x + \ldots + a_nx^n$, but even polynomials have useful forms adapted to another point: $a_0 + a_1(x-c) + \ldots + a_n(x-c)^n$, or even useful 'multiple center' forms: $a_0 + (x-c_1)(a_1 + \ldots + (x-c_n)(a_n]$.

(13) $y = 3x + 1$

(14) $y = \frac{4}{3}x + 2$

(15) Put the equation in slope-intercept form by adding $2y$ to both sides, subtracting 4 from both sides, and dividing by 2: $y = 3x - 2$, so the slope is 3 and the $y-$intercept is -2.

(16) Put the equation in slope-intercept form by subtracting $2x$ from both sides, and dividing by 5: $y = -\frac{2}{5}x + \frac{3}{5}$, so the slope is $-\frac{2}{5}$ and the $y-$intercept is $\frac{3}{5}$.

(17) Parallel lines have the same slope, so $y - 1 = 3(x - 1)$

(18) The equation of any non-vertical line containing the point $(2, -1)$ is $y - (-1) = m(x - 2)$. Parallel lines have the same slope, so $m = \frac{2-0}{3-2} = 2$. So the equation is $y - (-1) = 2(x - 2)$.

(19) The slope of any line perpendicular to a line with slope $m \neq 0$ is $-\frac{1}{m}$, the 'negative reciprocal' rule. So $y - 0 = -\frac{1}{3}(x - 1)$.

(20) To find the midpoint of two points and the bisector of the segment joining them, compute the simple average their horizontal and vertical coordinates respectively: $\frac{0+2}{2} = 1$ and $\frac{0+4}{2} = 2$ so the line goes through the point $(1, 2)$. The slope of the segment is $\frac{4-0}{2-0} = 2$, so the slope of any line perpendicular to it is $-\frac{1}{2}$ and the equation of the line with this slope through that point is $y - 2 = -\frac{1}{2}(x - 1)$.

(21) The slope of any line perpendicular to a vertical line $x = c$ is $m = 0$. So $y - 1 = 0$ or $y = 1$ whose graph is horizontal.

(22) The equation of any line perpendicular to a horizontal line $y = c$ is of the form $x = c$ and its slope is undefined. So $x = 2$.

(23) The line $2y - x = 4$ has slope $\frac{1}{2}$ so the equation of a line through the point $(1, 1)$ which is perpendicular to this line is $y - 1 = -2(x - 1)$. The intersection of these lines may be found by solving the latter for $y = -2x + 3$ and substituting into the equation of the first line: $2(-2x + 3) - x = 4$ so $x = \frac{2}{5}$ and $y = \frac{11}{5}$. By Pythagoras, this is the closest point on the line $2y - x = 4$ to the point $(1, 1)$ because the distance to any other point is the hypotenuse of a right triangle with one side being the segment between these points. This distance is $\sqrt{(\frac{2}{5} - 1)^2 + (\frac{11}{5} - 1)^2} = \frac{3\sqrt{5}}{5}$.

(24) The line $y = 2x - 3$ has slope 2 so the equation of a line through the point $(0, 1)$ which is perpendicular to this line is $y - 1 = -\frac{1}{2}(x - 0)$. The intersection of these lines may be found by substituting this into the equation of the first line: $-\frac{1}{2}x + 1 = 2x - 3$ so $x = \frac{8}{5}$ and $y = \frac{1}{5}$. The distance from $(0, 1)$ to this point, hence to the line, is This distance is $\sqrt{(\frac{8}{5} - 0)^2 + (\frac{1}{5} - 1)^2} = \frac{4\sqrt{5}}{5}$.

(25) The point $(0, 0)$ is on the line $y = 2x$. Both lines have slope 2 so the equation of a line through the point $(0, 0)$ which is perpendicular to the line $y = 2x + 3$ line is $y - 0 = -\frac{1}{2}(x - 0)$. The intersection of those lines may be found by substituting one into other: $-\frac{1}{2}x = 2x + 3$ so $x = -\frac{6}{5}$ and $y = \frac{3}{5}$. The distance from $(0, 0)$ to this point,

which is the shortest distance between point on one line and any point on the other, is $\sqrt{(-\frac{6}{5}-0)^2+(\frac{3}{5}-1)^2}=\frac{3\sqrt{5}}{5}$.

Solutions for Introduction to Polynomial Calculus

Section 2 Problems - The Slope of a Curve

Bob Palais

(1)

$$\frac{f(1+h)-f(1)}{h}=\frac{3(1+h)+2-(3(1)+2)}{h}=\frac{3h}{h}$$

which equals 3 for $h \neq 0$. The value which any *polynomial* expression in h approaches as h approaches 0 may be determined by setting h equal to 0. Note that before the h is removed from the denominator by finding an expression which is equivalent as long as $h \neq 0$, the expression is *not* a polynomial in h and cannot even be evaluated at $h = 0$.

In this case, the polynomial expression, 3, is a constant and does not even involve h. Evaluating the polynomial $p(h) = 3$ at $h = 0$ gives $p(0) = 3$, so this 'difference quotient' approaches 3 as h approaches 0. Since the curve $y = f(x)$ is a straight line with slope 3, we'd better hope that the slope of a curve computation reduces to the same slope as the line, and indeed it does. Since $f(1) = 5$, The tangent line at $(1, 5)$ is $y - 5 = 3(x - 0)$.

Note on the interpretation and manipulation of expressions of the form $f(x + h)$.: Many students interpret $f(x + h)$ purely symbolically and literally, symbolically replace any occurence of x with $x + h$. This is not a totally unreasonable idea since we teach to 'put what is in the parentheses whereever x is', but is correct in the context. For instance, if $f(x) = 4x$ one might incorrectly write $f(x + h) = 4x + h$, or if $g(x) = x^2$, one might incorrectly write $g(x + h) = x + h^2$. One 'systematic' way to avoid this would be always to replace x by what is between the parentheses *surrounded by parentheses.* In the above examples this would correctly give $f(x+h) = 4(x+h)$ and $g(x+h) = (x+h)^2$. The only problem is for 'simple' arguments in the parentheses it will give strange looking, yet not incorrect, extraneous parentheses, for example $f(a) = 4(a)$ or $g(3) = (3)^2$. You can easily remove these when you are sure they are not needed. An essentially equivalent conceptual approach is to understand the meaning of $f(x) = 4x$ as 'the function which multiplies its input (argument) by 4, so $f(x+h)$ says multiply $x+h$ by 4, and we know 4 times $x+h$ is $4(x + h) = 4x + 4h$ and not $4x + h$. Similarly $g(x) = x^2$ is the function which squares its input, so $g(x+h)$ is the $x+h$ squared, which is $(x+h)^2 = x^2 + 2xh + h^2$, and not $x+h^2$.

The following problems also use the above fact that $(x + h)^2 = x^2 + 2xh + h^2$, and $(x + h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$. These are special cases of the binomial rule

$$(x+h)^n=\sum_{j=0}^{n} C(n,j)x^{n-j}h^j$$

where $C(n, j)$ is the number of different ways of choosing j objects from a set of n objects when the order does not matter.

See http://www.math.utah.edu/∼palais/mst/Pascal.html for a flash application connecting different interpretations of $C(n, j)$ and demonstrating concretely the recursive formula known as Pascal's Triangle, $C(n, j) = C(n - 1, j - 1) + C(n - 1, j)$ and the direct formula for computing $C(n,j) = \frac{n!}{j!(n-j)!}$. (The symbol $n!$, spoken n factorial, represents the product of the positive integers less than or equal to n: $n! = 1 \cdot 2 \cdots n$.

One of the coolest and most powerful results accessible in the first year of calculus is the ability to generalize the binomial rule to the situation where n is not a positive integer, and develop analogous formulas for $\frac{1}{1+x} = (1+x)^{-1}$ and $\sqrt{1+x} = (1+x)^{1/2}$, etc.

(2)

$$\frac{f(0+h)-f(0)}{h} = \frac{h^2-0}{h} = \frac{h^2}{h}$$

which equals h for $h \neq 0$. Evaluating the polynomial $p(h) = h$ at $h = 0$ gives $p(0) = h$, so this 'difference quotient' approaches 0 as h approaches 0. The curve $y = f(x)$ is a parabola with its vertex pointing down at $(0,0)$ and by symmetry, we would expect its slope there would be 0 and indeed it does. The tangent line is horizontal: $y - 0 = 0(x - 0)$.

(3)

$$\frac{f(2+h)-f(2)}{h} = \frac{(2+h)^2-2^2}{h} = \frac{4+4h+h^2-4}{h} = \frac{4h+h^2}{h}$$

which equals $4+h$ for $h \neq 0$. Evaluating the polynomial $p(h) = 4+h$ at $h = 0$ gives $p(0) = 4$, so this 'difference quotient' approaches 4 as h approaches 0. The curve $y = f(x)$ is a parabola. Since $f(2) = 4$, The tangent line at $(2,4)$ is $y - 4 = 4(x-2)$.

(4)

$$\frac{f(1+h)-f(1)}{h} = \frac{(1+h)^2-3-(1^2-3)}{h} = \frac{1+2h+h^2-3-(1-3)}{h} = \frac{2h+h^2}{h}$$

which equals $2+h$ for $h \neq 0$. Evaluating the polynomial $p(h) = 2+h$ at $h = 0$ gives $p(0) = 2$, so this 'difference quotient' approaches 2 as h approaches 0. The curve $y = f(x)$ is a parabola. Since $f(1) = -2$, The tangent line at $(1,-2)$ is $y - (-2) = 2(x-1)$.

(5)

$$\frac{f(0+h)-f(0)}{h} = \frac{h^2+2h-1-(-1)}{h} = \frac{h^2+2h}{h}$$

which equals $h+2$ for $h \neq 0$. Evaluating the polynomial $p(h) = h+2$ at $h = 0$ gives $p(0) = 2$, so this 'difference quotient' approaches 2 as h approaches 0. The curve $y = f(x)$ is a parabola. Since $f(0) = -1$, The tangent line at $(0,-1)$ is $y - (-1) = 2(x-0)$.

(6)

$$\frac{f(1+h)-f(1)}{h} = \frac{3(1+h)^2-2-(3(1)^2-2)}{h} = \frac{3+6h+3h^2-2-(3-2)}{h} = \frac{6h+3h^2}{h}$$

which equals $6+3h$ for $h \neq 0$. Evaluating the polynomial $p(h) = 6+3h$ at $h = 0$ gives $p(0) = 6$, so this 'difference quotient' approaches 6 as h approaches 0. The curve $y = f(x)$ is a parabola. Since $f(1) = 1$, The tangent line at $(1,1)$ is $y - 1 = 6(x-1)$.

(7)

$$\frac{f(1+h)-f(1)}{h}=\frac{(1+h)^3-1^3}{h}=\frac{1+3h+3h^2+h^3-1)}{h}=\frac{3h+3h^2+h^3}{h}$$

which equals $3+3h+h^2$ for $h \neq 0$. Evaluating the polynomial $p(h)=3+3h+h^2$ at $h=0$ gives $p(0)=3$, so this 'difference quotient' approaches 3 as h approaches 0. Since $f(1)=1$, The tangent line at $(1,1)$ is $y-1=3(x-1)$.

(8)

$$\frac{f(0+h)-f(0)}{h}=\frac{h^3-0^3}{h}==\frac{h^3}{h}$$

which equals h^2 for $h \neq 0$. Evaluating the polynomial $p(h)=h^2$ at $h=0$ gives $p(0)=0$, so this 'difference quotient' approaches 0 as h approaches 0. Since $f(0)=0$, The tangent line at $(0,0)$ is $y-0=0(x-0)$.

(9)

$$\frac{f(x+h)-f(x)}{h}=\frac{(x+h)-x)}{h}=\frac{h}{h}$$

which equals 1 for $h \neq 0$. Evaluating the polynomial $p(h)=1$ at $h=0$ gives $p(0)=1$, so this 'difference quotient' approaches 1 as h approaches 0 for any value of x and $f'(x)=1$. Since the curve $y=f(x)$ is a straight line with slope 1, we'd better hope that the slope of a curve computation reduces to the same slope as the line, and indeed it does.

(10)

$$\frac{f(x+h)-f(x)}{h}=\frac{2(x+h)+5-(2x+5)}{h}=\frac{2h}{h}$$

which equals 2 for $h \neq 0$. Evaluating the polynomial $p(h)=2$ at $h=0$ gives $p(0)=2$, so this 'difference quotient' approaches 2 as h approaches 0 for any value of x and $f'(x)=2$. Since the curve $y=f(x)$ is a straight line with slope 2, we'd better hope that the slope of a curve computation reduces to the same slope as the line, and indeed it does.

(11)

$$\frac{f(x+h)-f(x)}{h}=\frac{3(x+h)^2-3x^2)}{h}=\frac{3x^2+6xh+3h^2-3x^2}{h}=\frac{6xh+3h^2}{h}$$

which equals $6x+3h$ for $h \neq 0$. Evaluating the polynomial $p(h)=6x+3h$ at $h=0$ gives $p(0)=6x$, so this 'difference quotient' approaches $6x$ as h approaches 0 for any value of x and $f'(x)=6x$. The curve $y=f(x)$ is a parabola, and it makes sense when $x>0$ to the right of the downward pointing vertes, the slope increases as x increases.

(12)

$$\frac{f(x+h)-f(x)}{h}=\frac{(x+h)^2-2(x+h)+3-(x^2-2x+3)}{h}$$

$$=\frac{x^2+2xh+h^2-2x-2h+3-x^2+2x-3}{h}=\frac{2xh+h^2-2h}{h}$$

which equals $2x+h-2$ for $h \neq 0$. Evaluating the polynomial $p(h) = 2x+h-2$ at $h=0$ gives $p(0) = 2x-2$, so this 'difference quotient' approaches $2x-2$ as h approaches 0 for any value of x and $f'(x) = 2x-2$.

(13)

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^3 - x^3}{h} = \frac{x^3+3x^2h+3xh^2+h^3-x^3}{h} = \frac{3x^2h+3xh^2+h^3}{h}$$

which equals $3x^2+3xh+h^2$ for $h \neq 0$. Evaluating the polynomial $p(h) = 3x^2+3xh+h^2$ at $h=0$ gives $p(0) = 3x^2$, so this 'difference quotient' approaches $3x^2$ as h approaches 0 for any value of x and $f'(x) = 3x^2$.

(14)

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^3+(x+h)^2-(x^3-x^2)}{h}$$

$$= \frac{x^3+3x^2h+3xh^2+h^3+x^2+2xh+h^2-x^3-x^2}{h} = \frac{3x^2h+3xh^2+h^3+2xh+h^2}{h}$$

which equals $3x^2+3xh+h^2+2x+h$ for $h \neq 0$. Evaluating the polynomial $p(h) = 3x^2+3xh+h^2+2x+h$ at $h=0$ gives $p(0) = 3x^2+2x$, so this 'difference quotient' approaches $3x^2+2x$ as h approaches 0 for any value of x and $f'(x) = 3x^2+2x$.

These examples should show you three patterns.

1. The derivative of the sum of functions will equal the sum of the derivatives:

If $f(x) = u(x)+v(x)$ then $f'(x) = u'(x)+v'(x)$. The aspects of the computation that always led to this did not have to do with the fact that the functions in the examples were polynomials.

2. The derivative of a constant multiple of a functions will equal the same constant multiple of its derivative:

If $f(x) = c(u(x))$ where c is a constant, then $f'(x) = c(u'(x))$. The aspects of the computation that always led to this did not have to do with the fact that the functions in the examples were polynomials.

3. The derivative of $f(x) = x^n$ is $f'(x) = nx^{n-1}$ which comes from the binomial rule, $(x+h)^n = x^n + nx^{n-1}h + \ldots$.

More solutions on the following page!!

(15) The point-slope form of a line containing the point $(-2, 4)$ is $y-4 = m(x-(-2))$, where m is the slope. Using the definition of a tangent line, $m = f'(-2)$ where $f(x) = x^2$, so $f'(x) = 2x$. Therefore, $m = 2(-2) = -4$ and the equation of the tangent line is $y - 4 = -4(x - (-2))$. Note that we only need to be given the x-value, -2, from which we could compute the corresponding y-value, $f(-2) = 4$. The given equation $y - 4 = -4(x - (-2))$ corresponds to the form given in the notes, $y - f(a) = f'(a)(x - a)$ with $f(x) = x^2$ and $a = -2$. Depending on the situation, you may or may not wish to 'simplify' $(x - (-2))$ to $x + 2$ because the first form exhibits the key information more clearly, and from this point of view, the latter form is not a 'simplification'.

(16) The point-slope form of a line containing the point $(2, -2)$ is $y-(-2) = m(x-2)$, where m is the slope. Using the definition of a tangent line, $m = f'(2)$ where $f(x) = x^2-3x$, so $f'(x) = 2x - 3$. Therefore, $m = 2(2) - 3 = 1$ and the equation of the tangent line is $y-(-2) = 1(x-2)$. Note that we only need to be given the x-value, 2, from which we could compute the corresponding y-value, $f(2) = -2$. The given equation $y - (-2) = 1(x - 2)$ corresponds to the form given in the notes, $y - f(a) = f'(a)(x - a)$ with $f(x) = x^2 - 3x$ and $a = 2$. Again, whether you choose to 'simplify' $(y - (-2))$ to $y + 2$ depends on the situation. Using '$+c$' may save an arithmetic operation in a computation, but $-(-c)$ may have more clarity.

Solutions for Introduction to Polynomial Calculus

Section 3 Problems - The Derivative of a Polynomial

Bob Palais

Calling the function in each problem $f(x)$ and using the three rules from the previous section:

The derivative of $f(x) = x^n$ is $f'(x) = nx^{n-1}$.

If $f(x) = u(x) + v(x)$ then $f'(x) = u'(x) + v'(x)$.

If $f(x) = c(u(x))$ where c is a constant, then $f'(x) = c(u'(x))$.

(1) $f'(x) = 9x^8$.

(2) $f'(x) = 100x^{49}$.

(3) $f'(x) = 3$.

(4) $f'(x) = 3x^2 - 2$.

(5) $f'(x) = 8x^3 + 3x^2 - 10x + 1$.

(6) $f'(x) = 11x^{10} - 18x^8 + 15$.

Computing $f'(x)$ and setting x equal to the x value at the given point on the graph:

(7) $f'(x) = 3x^2$, and $f'(1) = 3$ gives the slope of the curve at $(1, 1)$, as in problem (7) of the previous section. If you prefer when the function is given as $y = f(x)$ you may prefer to use $\frac{dy}{dx}$ (Leibniz notation) instead of $f'(x)$ (Newton notation). Then instead of $f'(1)$ we sometimes write $\frac{dy}{dx}|_{x=1}$ or even $\frac{dy}{dx}(1)$.

(8) $f'(x) = 2x$, and $f'(0) = 0$ gives the slope of the curve at $(0, 0)$, as in problem (2) of the previous section.

(9) $f'(x) = 3x^2 - 2x$, and $f'(1) = 1$ gives the slope of the curve at $(1, 0)$.

(10) $f'(x) = 4x^3 - 6x^2 + 5$, and $f'(2) = 13$ gives the slope of the curve at $(2, 7)$. The y-value comes from evaluating $f(2)$. The equation for the tangent line is $y - 7 = 13(x - 2)$.

(11) $f'(x) = 10x^9 - 5x^4$, and $f'(1) = 5$ gives the slope of the curve at $(1, 0)$. The y-value comes from evaluating $f(1)$. The equation for the tangent line is $y - 0 = 5(x - 1)$.

(12) $f'(x) = 2x - 2$, and $f'(x) = 0$ when $2x - 2 = 0$ or $x = 1$, $f'(x) > 0$ when $2x - 2 > 0$ or $x > 1$, and $f'(x) < 0$ when $2x - 2 < 0$ or $x < 1$. In words, the curve has positive slope for $x > 1$, negative slope for $x < 1$ and zero slope for $x = 1$.

(13) The (vertical) velocity of the ball t seconds after it is thrown is given by $\frac{ds}{dt} = s'(t) = -32t + 32$. The ball reaches its maximum height when its velocity changes from positive to negative, i.e., when $s'(t) = -32t + 32 = 0$ or $t = 1$. The height of the ball at $t = 1$ is $s(1) = 22$ feet.

(14) The (vertical) acceleration of the ball t seconds after it is thrown is given by $\frac{d^2s}{dt^2} = s'(t) = -32$ feet per second per second or feet per second squared. The velocity loses a constant 32 feet per second upward every second.

Solutions for Introduction to Polynomial Calculus

Section 4 Problems - Antiderivatives of Polynomials

Bob Palais

Calling the function in each problem $f(x)$ and using the three antidifferentiation rules corresponding to the previous three differentiation rules:

The antiderivative of $f(x) = x^n$ is $\int f(x)dx = \frac{x^{n+1}}{n+1} + C$.

If $f(x) = u(x) + v(x)$ then $\int f(x)dx = \int u(x)dx + \int v(x)dx$.

If $f(x) = c(u(x))$ where c is a constant, then $\int f(x)dx = c \int u(x)dx$.

(1) $\int f(x)dx = x^2 - 3x + C$. You should check this by taking its derivative!

(2) $\int f(x)dx = x^3 - 2x^2 + 5x + C$.

(3) $\int f(x)dx = \frac{x^6}{6} + \frac{x^4}{2} + x + C$.

(4) $\int f(x)dx = x^{10} - 4x^2 + C$.

Find the general antiderivative then impose the condition to determine C:

(5) $F(x) = \int f(x)dx = \frac{x^3}{3} - 5x + C$ and $F(0) = 2$ says $C = 2$, so $F(x) = \frac{x^3}{3} - 5x + 2$.

(6) $F(x) = \int f(x)dx = 2x^4 - x^2 + C$ and $F(1) = 4$ says $2 - 1 + C = 4$, so $C = 3$ and $F(x) = 2x^4 - x^2 + 3$.

(7) $F(x) = \int f(x)dx = \frac{x^4}{2} + C$ and $F(1) = 1$ says $\frac{1}{2} + C = 1$, so $C = \frac{1}{2}$ and $F(x) = \frac{x^4}{2} + \frac{1}{2}$.

(8) $F(x) = \int f(x)dx = \frac{x^4}{4} - \frac{x^2}{2} + C$ and $F(2) = 1$ says $4 - 2 + C = 1$, so $C = -1$ and $F(x) = \frac{x^4}{4} - \frac{x^2}{2} - 1$.

(9) The derivative of velocity is acceleration, and the acceleration of any body near the earth's surface under only the force of gravity is -32 feet per second squared. Since the (vertical) velocity is then the antiderivative of the acceleration,

$$v(t) = \int a(t)dt = \int -32dt = -32t + C$$

feet per second. We are given that $v(0) = 64$ feet per second, so $0 + C = 64$ and $v(t) = -32t + 64$ feet per second is the velocity after t seconds. The ball will achieve its maximum height when its vertical velocity changes from positive to negative, i.e., when $v(t) = -32t + 64 = 0$, so when $t = 2$ seconds.

(10) The derivative of (vertical) displacement, or height, is velocity, and the velocity of the ball is $v(t) = -32t + 64$ from the previous problem. Since the (vertical) displacement is then the antiderivative of the velocity,

$$s(t) = \int v(t)dt = \int -32t + 64dt = -16t^2 + 64t + C$$

feet. We are given that $s(0) = 6$ feet, so $0 + 0 + C = 6$ and $s(t) = -16t^2 + 64t + 6$ feet is the height of the ball after t seconds. Since the ball achieves its maximum height when $t = 2$ seconds, the maximum height it achieves is $s(2) = 70$ feet.