Math1210 Midterm 3 Summer, 2009 Dylan Zwick

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Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be **completely simplified**, unless otherwise stated, and in **exact form** (not approximated), unless otherwise stated. Place your answers in the designated space for each problem.

Potentially Useful Formulas

$$\frac{d(\tan(x))}{dx} = (sec(x))^2$$

$$\frac{d(\cot(x))}{dx} = -(csc(x))^2$$

$$\frac{d(sec(x))}{dx} = sec(x)\tan(x)$$

$$\frac{d(csc(x))}{dx} = -csc(x)\cot(x)$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

1. (15 pts) Let $y = 2x^3 - sec(\pi x) + \sqrt{x}$. If x changes from 1 to 1.035, approximately how much does y change? (You must use differentials to get the approximation.)

$$y'(x) = 6x^2 - \pi \sec(\pi x) \tan(\pi x) + \frac{1}{2\sqrt{x}}$$

$$y'(1) = 6(1^2) - \pi \sec(\pi) \tan(\pi) + \frac{1}{2\sqrt{1}}$$

$$= 6 - \pi(1)(0) + \frac{1}{2} = \frac{13}{2}$$

$$y(1) = 2(1^3) - \sec(\pi) + \sqrt{1} = 2 - (-1) + 1 = 4$$

$$dy = \frac{13}{2}(1.035 - 1) = .2275$$

$$y(1.035) \approx 4 + \frac{13}{2}(0.035) = 4.2275$$

Answer 1: $\Delta y \approx \boxed{dy = 0.2275}$

2. (10 points each) Evaluate

(a) $\int (5x^3(x^4-1)^{-2/3})\,dx$

$$\int (5x^3(x^4-1)^{-2/3})\,dx$$

$$= \frac{5}{4}\int 4x^3(x^4-1)^{-2/3}dx = \frac{5}{4}[3(x^4-1)^{1/3}] + C$$

$$= \frac{15}{4}(x^4-1)^{1/3} + C$$

Answer 2(a): $\frac{15}{4}(x^4-1)^{1/3} + C$

(b) $\int \left(3\sqrt[3]{t} - \frac{4}{t^3} + 5t^2 - \cos t + 2\right) dt$

$$= \frac{9}{4}t^{4/3} + \frac{2}{t^2} + \frac{5}{3}t^3 - \sin t + 2t + C$$

Answer 2(b): $\frac{9}{4}t^{4/3} + \frac{2}{t^2} + \frac{5}{3}t^3 - \sin t + 2t + C$

3. (15 points) Solve the following differential equation.

$$\frac{dy}{dx} = \frac{(x^2 - \sqrt{x})}{2y^3} \quad \text{such that} \quad y = -1 \quad \text{when} \quad x = 1$$

$$\int 2y^3 dy = \int (x^2 - \sqrt{x}) dx$$

$$\frac{y^4}{2} = \frac{x^3}{3} - \frac{2}{3} x^{3/2} + C$$

$$\Rightarrow \quad y = \pm \left[\frac{2x^3}{3} - \frac{4}{3} x^{3/2} + C \right]^{1/4}$$

$$y = - \left[\frac{2x^3}{3} - \frac{4}{3} x^{3/2} + C \right]^{1/4}$$

$$y(1) = - \left[\frac{2}{3} - \frac{4}{3} + C \right]^{1/4} = -1$$

$$C = \frac{5}{3}$$

$$y(x) = - \left[\frac{2x^3}{3} - \frac{4}{3} x^{3/2} + \frac{5}{3} \right]^{1/4}$$

Answer 3: $y = - \left[\frac{2x^3}{3} - \frac{4}{3} x^{3/2} + \frac{5}{3} \right]^{1/4}$

4. (5 points each) For $f(x)=\dfrac{x^2-2x-2}{x-3}$

(given also $f'(x)=\dfrac{x^2-6x+8}{(x-3)^2}$ and $f''(x)=\dfrac{2}{(x-3)^3}$)

(a) Find the x-values of the vertical asymptote(s), if they exist.

Vertical asymptote(s): $X = 3$

(b) Fill in the sign line for $f'(x)$

$$f'(x) = \frac{(x-2)(x-4)}{(x-3)^2}$$

	+		−		−		+	
<----		2		3		4		----> $f'(x)$

$f''(2) = -2$

$f''(4) = 2$

(c) Find all local minimum and maximum points, if they exist, or state that they DNE.

Local Max point(s): $(2,2)$

Local Min point(s): $(4,6)$

(d) Fill in the sign line for $f''(x)$

	−		+	
<----		3		----> $f''(x)$

(e) Find all inflection points, if they exist, or state that they DNE.

Inflection point(s): DNE

Note: As $f(x)$ is not defined at $x=3$, it is not the x-value of an inflection point.

5. (10 points) For the function $f(x)=\dfrac{4x-1}{x-4}$ on the closed interval [-1, 3], decide whether or not the Mean Value Theorem for Derivatives applies. If it does, find all possible values of c. If not, then state the reason.

$f(x)$ is continuous on $[-1,3]$, and

$f'(x) = \dfrac{4(x-4)-(4x-1)}{(x-4)^2} = -\dfrac{15}{(x-4)^2}$ ⇐ defined on $(-1,3)$

So, the MVT applies.

$f(3) = -11$

$f(-1) = 1$

$\dfrac{f(3)-f(-1)}{3-(-1)} = \dfrac{-11-1}{4} = -3$

$f'(c) = -3 = -\dfrac{15}{(c-4)^2}$

$(c-4)^2 = 5$

$\Rightarrow c-4 = \pm\sqrt{5} \Rightarrow c = 4\pm\sqrt{5}$

in $(-1,3] \Rightarrow c = 4-\sqrt{5}$

MVT applies: True or False (circle one)

If true, then c = $4-\sqrt{5}$

If false, then why? NA

6. (7 points) Evaluate this integral. $\int \dfrac{(2x+3)^2}{\sqrt{x}}\,dx$

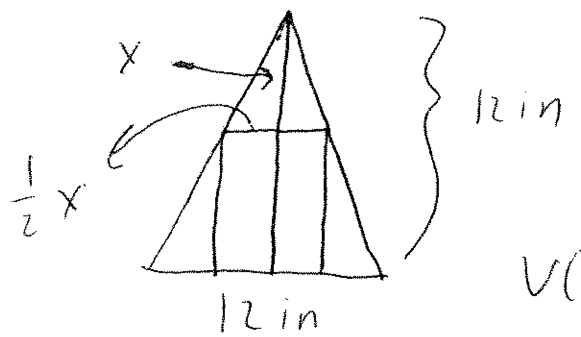
$= \int \dfrac{4x^2+12x+9}{\sqrt{x}}\,dx = \int\left(4x^{3/2}+12x^{1/2}+\dfrac{9}{\sqrt{x}}\right)dx$

$= \dfrac{8}{5}x^{5/2} + 8x^{3/2} + 18\sqrt{x} + C$

Answer: $\boxed{\dfrac{8}{5}x^{5/2}+8x^{3/2}+18\sqrt{x}+C}$

7. (15 points) Find the dimensions of the right circular cylinder of greatest volume that can be inscribed in a right circular cone, if the cone has a radius of 6 inches and height of 12 inches.

(Note: The volume of a cylinder is $V = \pi r^2 h$; the volume of a cone is $V = \frac{1}{3}\pi r^2 h$.)



$$V(x) = \pi \left(\frac{1}{2}x\right)^2 (12 - x)$$

$$V(x) = \frac{\pi x^2}{4}(12 - x)$$

$$= 3\pi x^2 - \frac{\pi x^3}{4}$$

$$V'(x) = 6\pi x - \frac{3}{4}\pi x^2$$

$$V''(x) = 6\pi - \frac{3}{2}\pi x$$

$V'(x) = 0 \Rightarrow \frac{3}{4}\pi x^2 = 6\pi x$ $x \neq 0$ (Volume here is obviously 0)

$$\Rightarrow \frac{3}{4}\pi x = 6\pi$$

$$\Rightarrow 3x = 24$$

$$x = 8$$

$\Rightarrow$ height of cylinder $= 12 - x = 12 - 8 = 4$ in

radius of cylinder $= \frac{1}{2}x = 4$ in

Volume of cylinder $= \pi (4\text{in})^2 (4\text{in}) = 64\pi$ in^3

Answer 7: $h = 4$in, $r = 4$in, $V = 64\pi$ in^3

Note: Any answer that arrived at this right answer correctly got full credit even if, for example, the answer is just $V = 64\pi$ in^3.

8. (15 points) Evaluate the definite integral **using the definition** (the tedious way).

$\int_0^2 3x^2 dx$. (Note: Here is the definition. $\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$)

$\Delta x =$ $\frac{2-0}{n} = \frac{2}{n}$ (2 points)

$x_i =$ $\left(\frac{2}{n}\right) i$ (2 points)

(Just fill in the formula here, and do the simplifying after this.)

$\sum_{i=1}^{n} f(x_i)\Delta x =$ $\sum_{i=1}^{n} 3\left(\frac{2}{n} i\right)^2 \cdot \left(\frac{2}{n}\right)$ (3 points)

$$= \sum_{i=1}^{n} \frac{24 i^2}{n^3} = \cancel{24(}\cdot \frac{24n(n+1)(2n+1)}{6n^3}$$

$$= \frac{4n(n+1)(2n+1)}{n^3} = \frac{4n(2n^2+3n+1)}{n^3}$$

$$= \frac{8n^3+12n^2+4n}{n^3} = 8 + \frac{12}{n} + \frac{4}{n^2}$$

$$\lim_{n\to\infty}\left(8 + \frac{12}{n} + \frac{4}{n^2}\right) = 8$$

$\int_0^2 3x^2 dx =$ 8 (8 points)