Math1210 Midterm 3 Summer, 2009 Dylan Zwick

Name ______________________________ Date _____________

Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be **completely simplified**, unless otherwise stated, and in **exact form** (not approximated), unless otherwise stated. Place your answers in the designated space for each problem.

Potentially Useful Formulas

$$\frac{d(\tan(x))}{dx} = (sec(x))^2$$

$$\frac{d(\cot(x))}{dx} = -(csc(x))^2$$

$$\frac{d(sec(x))}{dx} = sec(x)\tan(x)$$

$$\frac{d(csc(x))}{dx} = -csc(x)\cot(x)$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

1. (15 pts) Let $y = 2x^3 - sec(\pi x) + \sqrt{x}$. If x changes from 1 to 1.035, approximately how much does y change? (You must use differentials to get the approximation.)

Answer 1: ____________________________________

2. (10 points each) Evaluate

(a) $\int \left(5x^3(x^4-1)^{-2/3}\right)dx$

Answer 2(a): ______________________________

(b) $\int \left(3\sqrt[3]{t}-\frac{4}{t^3}+5t^2-\cos t+2\right)dt$

Answer 2(b): ______________________________

3. (15 points) Solve the following differential equation.

$$\frac{dy}{dx} = \frac{(x^2 - \sqrt{x})}{2y^3} \quad \text{such that} \quad y = -1 \quad \text{when} \quad x = 1$$

Answer 3:____

4. (5 points each) For $f(x)=\dfrac{x^2-2x-2}{x-3}$

(given also $f'(x)=\dfrac{x^2-6x+8}{(x-3)^2}$ and $f''(x)=\dfrac{2}{(x-3)^3}$)

(a) Find the x-values of the vertical asymptote(s), if they exist.

Vertical asymptote(s): ________________________________

(b) Fill in the sign line for $f'(x)$

<--> $f'(x)$

(c) Find all local minimum and maximum points, if they exist, or state that they DNE.

Local Max point(s): ________________________________

Local Min point(s): ________________________________

(d) Fill in the sign line for $f''(x)$

<--> $f''(x)$

(e) Find all inflection points, if they exist, or state that they DNE.

Inflection point(s): ________________________________

5. (10 points) For the function $f(x) = \frac{4x-1}{x-4}$ on the closed interval [-1, 3], decide whether or not the Mean Value Theorem for Derivatives applies. If it does, find all possible values of c. If not, then state the reason.

MVT applies: True or False (circle one)

If true, then c = ____________________

If false, then why? ____________________

6. (7 points) Evaluate this integral. $\int \frac{(2x+3)^2}{\sqrt{x}}\,dx$

Answer:____________________

7. (15 points) Find the dimensions of the right circular cylinder of greatest volume that can be inscribed in a right circular cone, if the cone has a radius of 6 inches and height of 12 inches.

(Note: The volume of a cylinder is $V = \pi r^2 h$; the volume of a cone is $V = \frac{1}{3}\pi r^2 h$.)

Answer 7: ____

8. (15 points) Evaluate the definite integral **using the definition** (the tedious way).

$\int_0^2 3x^2\,dx$. (Note: Here is the definition. $\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$)

Δx = ____________________________

x_i = ______________________________

(Just fill in the formula here, and do the simplifying after this.)

$\sum_{i=1}^{n} f(x_i)\,\Delta x$ = ____

$\int_0^2 3x^2\,dx$ = __________________________