Math1210 Midterm 2 Summer, 2009 Dylan Zwick

Name Dylan Zwick (Solutions) Date 5/29/09

Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. For $y = 3\cos(5(x - \frac{\pi}{2})) - 1$, answer the following questions.

(a) (5 pts) What is the amplitude? 3

(b) (5 pts) What is the period? $\frac{2\pi}{5}$

(c) (5 pts) What is the vertical shift? 1 units

up or down (circle one)

(d) (5 pts) What is the horizontal shift? $\pi/2$ units

left or right (circle one)

(e) (10 pts) Fill in the table with the y-values corresponding to the given x-values. (Note: I want exact values here, with your work shown...so don't just plug these in on your calculator and give me an approximation.)

x	y
$\frac{\pi}{2}$	2
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{-3\sqrt{2}-2}{2}$
$\frac{11\pi}{15}$	$\frac{-3\sqrt{3}-2}{2}$

2. (15 pts) Find the limit, if it exists. (Show all work.)

$$\lim_{x\to 0} \frac{\cos(2x)\sin(4x)csc(5x)}{sec(3x)}$$

$$\lim_{x\to 0} \frac{\cos(2x)\sin(4x)\cdot\left(\frac{1}{\sin(5x)}\right)}{\frac{1}{\cos(3x)}} = \lim_{x\to 0} \frac{\cos(2x)\left(\frac{4\sin(4x)}{4x}\right)\left(\frac{1}{\frac{5\sin(5x)}{5x}}\right)}{1/\cos(3x)}$$

$$= \frac{1\cdot(4)\cdot\left(\frac{1}{5}\right)}{1/1} = \frac{4}{5}$$

Answer 2: $\boxed{\frac{4}{5}}$

3. (10 pts each) Find the indicated derivative of the given functions.

(a) $D_x(\tan(4x^2+5x-1)\cos^2(3x))$ (Do **not** bother to simplify!)

$$\tan(4x^2+5x-1)\,2\cos(3x)\cdot(-\sin(3x))\cdot 3$$
$$+\cos^2(3x)\sec^2(4x^2+5x-1)\cdot(8x+5)$$

Answer 3(a): $(8x+5)\cos^2(3x)\sec^2(4x^2+5x-1) - 6\cos(3x)\sin(3x)\tan(4x^2+5x-1)$

(Note: This is #3 continued!)

(b) $\frac{d}{dx}\left(\frac{x^4-3x^2+1}{x^3-\sqrt[4]{x}}\right)^5$ (Do **not** bother to simplify!)

$$5\left(\frac{x^4-3x^2+1}{x^3-x^{1/4}}\right)^4\cdot\left(\frac{(x^3-x^{1/4})\cdot(4x^3-6x)-(x^4-3x^2+1)\cdot\left(3x^2-\frac{1}{4x^{3/4}}\right)}{(x^3-x^{1/4})^2}\right)$$

Answer 3(b): $\boxed{5\left(\frac{x^4-3x^2+1}{x^3-x^{1/4}}\right)^4\cdot\left(\frac{(x^3-x^{1/4})(4x^3-6x)-(x^4-3x^2+1)\cdot(3x^2-\frac{1}{4}x^{-3/4})}{(x^3-x^{1/4})^2}\right)}$

(c) $f'(1)$ if $f(x)=(2x-\frac{1}{x})^3(4x^3-2)^4$

$$f'(x)=(2x-\tfrac{1}{x})^3 4(4x^3-2)^3\cdot(12x^2)$$
$$+(4x^3-2)^4\cdot 3(2x-\tfrac{1}{x})^2\cdot(2+\tfrac{1}{x^2})$$

$$f'(x)=(2x-\tfrac{1}{x})^3(48x^2(4x^3-2)^3)+3(4x^3-2)^4(2x-\tfrac{1}{x})^2(2+\tfrac{1}{x^2})$$

$$f'(1)=(2-1)^3\cdot(48\,1^2(4(1^3)-2)^3)+3(4(1^3)-2)^4(2(1)-\tfrac{1}{1})^2(2+\tfrac{1}{1^2})$$
$$=1^3\cdot 48\cdot 8+3\cdot 2^4\cdot 1^2\cdot 3$$
$$=48\cdot 8+3\cdot 16\cdot 3=528$$

Answer 3(c): $\boxed{528}$

(Note: This is #3 continued!)

(d) $\frac{dy}{dx}$ given $2x^4 y + y^3 = 2x^2 - 6x$

(Solve for $\frac{dy}{dx}$, i.e. get it by itself, but don't bother simplifying any further.)

$$\frac{d}{dx}(2x^4y + y^3) = \frac{d}{dx}(2x^2 - 6x)$$

$$\Rightarrow \quad 2x^4y' + 8x^3y + 3y^2y' = 4x - 6$$

$$\Rightarrow (2x^4 + 3y^2)y' = 4x - 6 - 8x^3y$$

$$\Rightarrow \quad y' = \frac{4x - 6 - 8x^3y}{2x^4 + 3y^2}$$

Answer 3(d): $\frac{dy}{dx} = \frac{4x - 6 - 8x^3y}{2x^4 + 3y^2}$

(e) $f'''(x)$ for $f(x) = (3x-4)^{\frac{2}{5}}$

$$f'(x) = \frac{2}{5}(3x-4)^{-3/5} \cdot 3 = \frac{6}{5}(3x-4)^{-3/5}$$

$$f''(x) = -\frac{18}{25}(3x-4)^{-8/5} \cdot 3 = -\frac{54}{25}(3x-4)^{-8/5}$$

$$f'''(x) = -\frac{54}{25}\left(-\frac{8}{5}\right) \cdot (3x-4)^{-13/5} \cdot 3$$

$$= \frac{1296}{125}(3x-4)^{-13/5}$$

Answer 3(e): $\frac{1296}{125}(3x-4)^{-\frac{13}{5}}$

4. (15 points) Choose either A or B to do. You CANNOT get credit for both and we will not choose for you, if you decide to try both of them.

Grade A or B ***(circle one)***

A. A metal rod has the shape of a right circular cylinder. As it is being heated, its length is increasing at a rate of 0.005 cm/min and its radius is increasing at 0.001 cm/min. At what rate is the volume changing when the rod has length 40 cm and radius 1.5 cm?

B. A softball diamond has the shape of a square with sides 40 ft. long. If a player is running from second base to third base at a speed of 20 ft/sec, at what rate is her distance from home plate changing when she is 30 ft from third base?

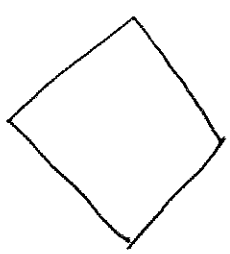
A. $V = \pi r^2 h$

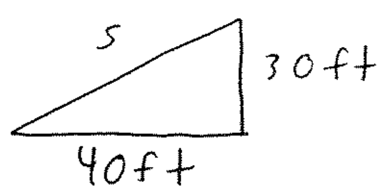
$$\frac{dV}{dt} = 2\pi r h\left(\frac{dr}{dt}\right) + \pi r^2\left(\frac{dh}{dt}\right)$$

$$= 2\pi(1.5\,\text{cm})(40\,\text{cm})(.001\ \text{cm/min}) + \pi(1.5\,\text{cm})^2 \cdot (0.005\ \text{cm/min})$$

$$= \boxed{0.13125\pi\ \text{cm}^3/\text{min}}$$

B.





$$s^2 = x^2 + y^2$$

$$2s\frac{ds}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$\Rightarrow s\frac{ds}{dt} = y\frac{dy}{dt} \quad \text{as } \frac{dx}{dt} = 0$$

$$\Rightarrow \frac{ds}{dt} = \left(\frac{y}{s}\right)\frac{dy}{dt}$$

$$= \frac{30}{\sqrt{40^2+30^2}} \cdot (-20) = -12\ \text{ft/s}$$

Answer 4A: $0.13125\pi\ \text{cm}^3/\text{min}$ OR Answer 4B: $-12\ \text{ft/s}$

Extra Credit (7 pts) :
Show that the tangent lines to the curves $y^2=4x^3$ and $2x^2+3y^2=14$ at (1, 2) are perpendicular to each other.

$$\frac{d}{dx}(y^2)=\frac{d}{dx}(4x^3) \qquad \frac{d}{dx}(2x^2+3y^2)=\frac{d}{dx}(14)$$

$$\Rightarrow 2y\,y'=12x^2 \qquad \Rightarrow 4x+6yy'=0$$

$$\Rightarrow y'=\frac{6x^2}{y} \qquad \Rightarrow y'=-\frac{2x}{3y}$$

$$y'(1,2)=\frac{6(1^2)}{2}=3 \qquad y'(1,2)=\frac{-2(1)}{3(2)}=-\frac{1}{3}$$

The slope of 3 is perpendicular to $-\frac{1}{3}$.