Math1210 Midterm 1 Summer, 2009 Dylan Zwick

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Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. (5 points each) Find each limit, if it exists.

(a) $\lim_{x \to -1} (-3x^4 - 2x^3 + 5x^2 + 4)$

$-3(-1)^4 - 2(-1)^3 + 5(-1)^2 + 4$

$= -3 + 2 + 5 + 4 = 8$

Answer 1(a): $\boxed{8}$

(b) $\lim_{x \to 3} \dfrac{2x^2 - 11x + 15}{x - 3}$

$\lim_{x \to 3} \dfrac{2x^2 - 11x + 15}{x - 3} = \lim_{x \to 3} \dfrac{(2x-5)(x-3)}{(x-3)} = \lim_{x \to 3} (2x-5)$

Answer 1(b): $\boxed{1}$

(c) $\lim_{x \to -\infty} \dfrac{6x^5 + 7x^3 - 3x^2}{1 + 2x - 3x^5}$

$\lim_{x \to -\infty} \dfrac{6x^5 + 7x^3 - 3x^2}{1 + 2x - 3x^5} = \lim_{x \to -\infty} \dfrac{6 + \frac{7}{x^2} - \frac{3}{x^3}}{\frac{1}{x^5} + \frac{2}{x^4} - 3} = -2$

Answer 1(c): $\boxed{-2}$

(d) $\lim_{x \to \infty} \dfrac{3x^3 + 8x + 1}{3x^2 - 4}$

$\lim_{x \to \infty} \dfrac{3x^3 + 8x + 1}{3x^2 - 4} \cdot \dfrac{\left(\frac{1}{x^2}\right)}{\left(\frac{1}{x^2}\right)} = \lim_{x \to \infty} \dfrac{3x + \frac{8}{x} + \frac{1}{x^2}}{3 - 4/x^2} = \lim_{x \to \infty} \dfrac{3x}{3}$

$= \lim_{x \to \infty} x = \infty$

Answer 1(d): $\boxed{\infty}$

2. (5 points each) A rocket is shot straight up so that its position after t seconds is given by
$s(t) = -16t^2 + 20t + 25$ (measured in feet).

(a) Find the velocity for any time t.

$s'(t) = -32t + 20$

Answer 2(a): $\boxed{v(t) = -32t + 20}$

(b) Find the acceleration for any time t.

$v'(t) = -32$

Answer 2(b): $\boxed{a(t) = -32}$

(c) When does the rocket reach its maximum height?

$-32t + 20 = 0 \Rightarrow t = \frac{20}{32} = \frac{5}{8}$

$s\left(\frac{5}{8}\right) = -16\left(\frac{5}{8}\right)^2 + 20\left(\frac{5}{8}\right) + 25$

Answer 2(c): $\boxed{\frac{5}{8}\ s}$

3. (10 points) Find the equation of the tangent line to the curve (Me)
$f(x) = 4x^2 - 5x + 2$ at $x = 1$.

$f'(x) = 8x - 5$

$f'(1) = 8(1) - 5 = 3 \leftarrow$ slope 3.

$f(1) = 4(1^2) - 5(1) + 2 = 1$

$(1, 1)$ is the point on the curve when $x = 1$.

$\Rightarrow (y - 1) = 3(x - 1) \Leftrightarrow y = 3x - 2$

Answer 3: $\boxed{y = 3x - 2}$

4. (5 points) Find the derivative of $y=x^6-3x^5+4x^3-2x^2+9x+5$

$y' = 6x^5 - 15x^4 + 12x^2 - 4x + 9$

Answer 4: $\boxed{y' = 6x^5 - 15x^4 + 12x^2 - 4x + 9}$

5. (5 points) Evaluate the integral $Y=\int(10x^4-4x^3+6x^2+3)\,dx$ given that $Y=5$ when $x=1$.

$\int (10x^4 - 4x^3 + 6x^2 + 3)\,dx = 2x^5 - x^4 + 2x^3 + 3x + C$

$2(1)^5 - (1)^4 + 2(1)^3 + 3(1) + C = 5$

$2 - 1 + 2 + 3 + C = 5$

$\Rightarrow C = -1 \qquad \Rightarrow Y = 2x^5 - x^4 + 2x^3 + 3x - 1$

Answer 5: $\boxed{2x^5 - x^4 + 2x^3 + 3x - 1}$

6. (5 points each) For $f(x)=\dfrac{(2x-5)(3x^2-2x-8)}{6x^2-7x-20}$ (Me)

(a) For what x-values is $f(x)$ discontinuous?

$6x^2 - 7x - 20 = (3x+4)(2x-5)$

$x = -\frac{4}{3}, \frac{5}{2}$

Answer 6(a): $x = \{-\frac{4}{3}, \frac{5}{2}\}$

(b) How should $f(x)$ be defined to make it continuous at its discontinuities? That is, redefine $f(x)$ as a piecewise, continuous function.

$(3x^2 - 2x - 8) = (3x+4)(x-2) = (3x+4)(x-2)$

$f(x) = \dfrac{(2x-5)(3x^2-2x-8)}{6x^2-7x-20} = \dfrac{(2x-5)(3x+4)(x-2)}{(3x+4)(2x-5)} = (x-2)$

$f(x) = \begin{cases} \dfrac{(2x-5)(3x^2-2x-8)}{6x^2-7x-20} & x \neq -\frac{4}{3}, \frac{5}{2} \\ \end{cases}$

Answer 6(b): $\boxed{f(x) = \begin{cases} \dfrac{(2x-5)(3x^2-2x-8)}{6x^2-7x-20} & x \neq -\frac{4}{3}, \frac{5}{2} \\ -\frac{10}{3} & x = -\frac{4}{3} \\ \frac{1}{2} & x = \frac{5}{2} \end{cases}}$

(Me) 7. (10 points) Use the definition of the derivative, namely $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$, to find the derivative for $f(x)=\frac{-5}{x^3}$.

$$f(x+h)=\frac{-5}{(x+h)^3} \qquad f(x)=-\frac{5}{x^3}$$

$$f'(x)=\lim_{h\to 0}\frac{\frac{-5}{(x+h)^3}-\left(-\frac{5}{x^3}\right)}{h}$$

$$=\lim_{h\to 0}\frac{\frac{-5x^3+5(x+h)^3}{(x+h)^3x^3}}{h}$$

$$=\lim_{h\to 0}\frac{\frac{15x^2h+15xh^2+5h^3}{(x+h)^3x^3}}{h}$$

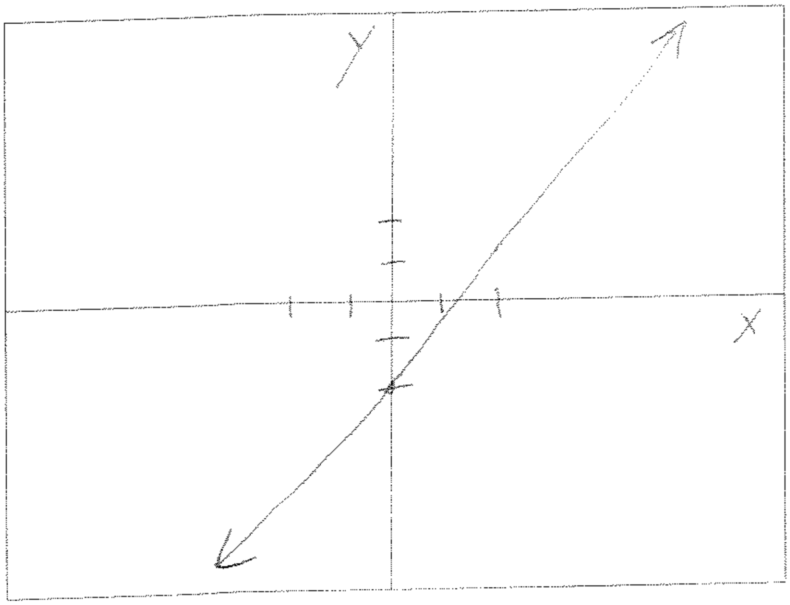
$$=\lim_{h\to 0}\frac{15x^2+15xh+5h^2}{(x+h)^3x^3}$$

$$=\frac{15x^2}{x^6}=\frac{15}{x^4}$$

Answer 7: $\boxed{f'(x)=\frac{15}{x^4}}$

8. (4 points each) For line L given by $3x - 2y = 4$,

(a) Graph the line L.



$2y = 3x - 4$

$y = \frac{3}{2}x - 2$

(b) What is the slope of the line L? Slope of L = $3/2$

(c) What is the slope of any parallel line to L?

Slope of parallel line = $3/2$

(d) What is the slope of any line perpendicular to L?

Slope of perpendicular line = $-2/3$

(e) What is the equation of the line going through the point (5, -1) and perpendicular to line L?

$$(y+1) = -\frac{2}{3}(x-5)$$

$$y + 1 = -\frac{2}{3}x + \frac{10}{3}$$

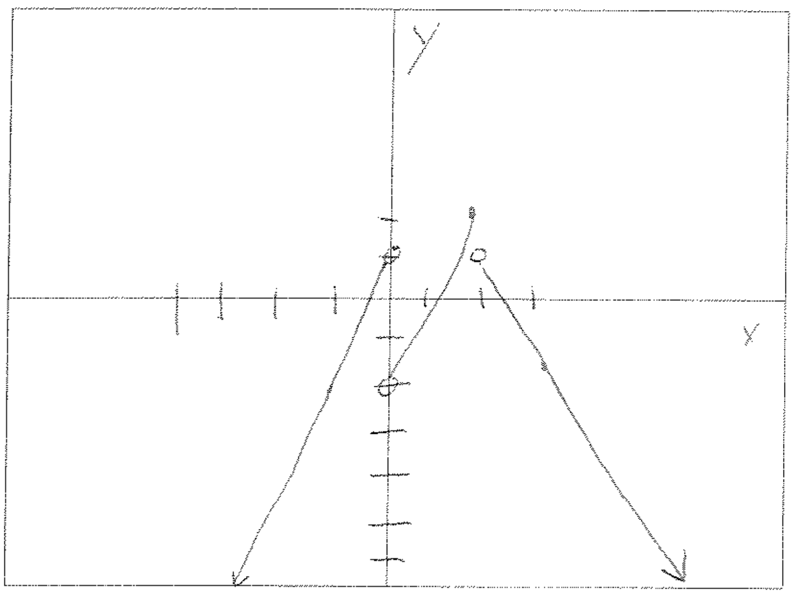
$$y = -\frac{2}{3}x + \frac{7}{3}$$

Equation of line: $\boxed{y = -\frac{2}{3}x + \frac{7}{3}}$

9. (20 points total) For this piecewise function.

$$g(x)=\begin{cases} 3x+1 & -4 \le x < 0 \\ x^2-2 & 0 < x \le 2 \\ -2x+5 & x > 2 \end{cases}$$

(a) Graph this function.



Now, evaluate the following, or state that the answer does not exist.

(b) $\lim_{x \to 2} g(x)$ = Does not exist

(c) $\lim_{x \to 5} g(x)$ = -5

(d) $\lim_{x \to 0^+} g(x)$ = -2

(e) $\lim_{x \to 0^-} g(x)$ = 1

(f) $g(2)$ = 2

(g) $g(0)$ = Does not exist (undefined)

10. (15 points total) Find the derivatives of the following functions. (Me)
Don't bother to simplify!

(a) $f(x)=\dfrac{2x^{-3}+4x^{2}-7x}{5x^{5}-3x^{3}+2x^{2}-1}$

Quotient Rule:

$$\frac{(5x^5-3x^3+2x^2-1)(-6x^{-4}+8x-7)-(2x^{-3}+4x^2-7x)(25x^4-9x^2+4x)}{(5x^5-3x^3+2x^2-1)^2}$$

$f'(x) = \boxed{\dfrac{(5x^5-3x^3+2x^2-1)(-6x^{-4}+8x-7)-(2x^{-3}+4x^2-7x)(25x^4-9x^2+4x)}{(5x^5-3x^3+2x^2-1)^2}}$

(b) $f(x)=(4x^{3}+7x^{2}-x)(5x^{4}+8x^{5})$

$f'(x) = \boxed{(4x^3+7x^2-x)(20x^3+40x^4)+(12x^2+14x-1)(5x^4+8x^5)}$

Extra Credit: (6 points) Describe/define, in your own words, what a linear operator is. Then, name one linear operator and give examples of how its linearity is used in mathematics. (Me)

Definition: A linear operator is an an operator (a map) that satisfies:

$$L(aX+bY)=aL(X)+bL(Y)$$

My linear operator: The derivative

Examples of my linear operator being used:

$D(2x^2-3x)=2D(x^2)-3D(x)=4x-3$