

Finding Groebner Bases with Maple

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Friday, June 20th, 2008

Today we're just going to cover how to use Maple to calculate Groebner bases, and then work out some examples from class and from the textbook. The current plan is that next week starting on Monday we'll be diving into creating our own Groebner bases program using Maple or, if you're daring, C++.

1 Using the Groebner Bases Package

First thing you'll want to do is open up Maple. It should be under applications (I think) on your computer. In any case, it will be the program with a maple leaf on it.

Now, once you've opened up maple the first thing you'll want to type is:

```
with(Groebner);
```

which will set up all the Groebner bases functions. Please note that any and all commands in maple must be followed by a semicolon, kind of like each line of C++ code.

Now, the two major commands that we'll want to use with this package are **normalf** and **gbasis**. The first command calculates the remainder of a polynomial after division by a set of other polynomials. The second command calculates a Groebner basis for a set of polynomials.

1.1 The normalf command

So, let's get right into these and see how these work. Type this command into Maple:

```
normalf( $x^3 + 3y^2$ , [ $x^2 + y, x + 2xy$ ], tdeg(x,y));
```

This will return the result of dividing the polynomial $x^3 + 3y^2$ by the polynomials $x^2 + y$ and $x + 2xy$, in that order, using graded reverse lex order. To use lex order you'd type in the same command, only instead of **tdeg**(**x,y**) you'd use **plex**(**x,y**).

Try doing this with some of the examples from the book. Namely, try dividing the polynomial $f = x^2y + xy^2 + y^2$ by the polynomials $f_1 = xy - 1$ and $f_2 = y^2 - 1$. Do this for both possible orderings of the polynomials, and verify that you get a different answer depending on the order. Use lex order with $x > y$.

1.2 The gbasis command

The **gbasis** command is used to calculate a Groebner basis for a set of polynomials. So, type the following into the command prompt:

```
gbasis([ $x^2 + y, 2xy + y^2$ ], plex(x,y));
```

This will calculate a Groebner basis for the two polynomials using lex order with $x > y$.

Use this function to calculate the Groebner basis for the ideal generated by the polynomials $\langle y - x^2, z - x^3 \rangle$ using lex order with $x > y > z$ and with $y > z > x$ and verify that the Groebner basis you get with the second ordering is more simple.

Finally, investigate the Groebner basis for the ideal generated in our study of the tangent surface of the twisted cubic, and try to regenerate the Groebner basis listed on page 131 of the textbook.

If you're feeling particularly adventurous, you can take a look at the three dimensional object that Prof. Bertram talked about when you also look at the acceleration vector. However, be warned, I tried this and the Groebner basis was not calculated after almost two hours!