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Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. (10 points each) Evaluate the following integrals.

$$(a) \int_0^{\ln 3} \int_0^1 xy e^{xy^2} dy dx$$

$$u = xy^2$$

$$du = 2xy dy$$

$$\Rightarrow \int_0^{\ln 3} \int_0^x \frac{1}{2} e^u du dx$$

$$= \int_0^{\ln 3} \left[\frac{1}{2} e^u \Big|_0^x \right] dx$$

$$\int_0^{\ln 3} \left[\frac{1}{2} e^x - \frac{1}{2} \right] dx$$

$$= \frac{1}{2} (e^x - x) \Big|_0^{\ln 3}$$

$$= \frac{1}{2} (3 - \ln 3) - \frac{1}{2} (1)$$

$$= \boxed{1 - \frac{1}{2} \ln 3}$$

Answer: 1 - \frac{1}{2} \ln 3

$$(b) \int_0^2 \int_0^1 (7-y) dx dy$$

$$= \int_0^2 \left[(7x - yx) \Big|_0^1 \right] dy$$

$$= \int_0^2 (7-y) dy$$

$$= 7y - \frac{y^2}{2} \Big|_0^2$$

$$= \left(7(2) - \frac{2^2}{2} \right) - \left(7(0) - \frac{0^2}{2} \right)$$

$$= 14 - 2$$

$$= 12$$

Answer: 12

$$(c) \int_0^1 \int_0^1 \frac{y}{(xy+1)^2} dx dy .$$

$$u = xy+1$$

$$du = y dx$$

$$\Rightarrow \int_0^1 \int_1^{y+1} \frac{du}{u^2} du dy$$

$$= \int_0^1 \left(-\frac{1}{u} \Big|_1^{y+1} \right) dy$$

$$= \int_0^1 \left(-\frac{1}{y+1} - \left(-\frac{1}{1} \right) \right) dy$$

$$= \int_0^1 \left(1 - \frac{1}{y+1} \right) dy = y - \ln(y+1) \Big|_0^1$$

$$= 1 - \ln 2$$

Answer: 1 - \ln 2

2. (10 points) Find the volume of the solid in the first octant enclosed by $z=4-x^2$ and $y=2$.

The region R in the xy -plane is $\{(x,y) | 0 \leq y \leq 2, 0 \leq x \leq 2\}$

$$\Rightarrow V = \int_0^2 \int_0^2 (4-x^2) dy dx$$

$$= \int_0^2 (4y - x^2 y \Big|_0^2) dx = \int_0^2 (8 - 2x^2) dx$$

$$= 8x - \frac{2}{3}x^3 \Big|_0^2 = 16 - \frac{16}{3} = \boxed{\frac{32}{3}}$$

Answer: $\frac{32}{3}$