Math2210 Quiz 8 (Section 12.8) Summer, 2008 Dylan Zwick

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Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. *(15 points)* Find all critical points for $f(x,y)=\cos x+\cos y+\cos(x+y)$ (assume $x, y\in(0,\pi)$). Indicate whether each point is a max, min or saddle point.

cosine is always differentiable, and $x,y \in (0,\pi)$ is an open set, so the only critical points are when $\nabla f(x,y) = \vec{0}$.

$$\nabla f(x,y) = \langle -\sin x - \sin(x+y), -\sin y - \sin(x+y) \rangle$$

Now, if $-\sin x - \sin(x+y) = 0 = -\sin y - \sin(x+y)$

$\Rightarrow \sin x = \sin y \Rightarrow x = y$ or $x = y + \frac{\pi}{2}$ if $x < \frac{\pi}{2}$.

If $x = y$ we have

$$-\sin x - \sin 2x = 0 \Rightarrow \sin x = -\sin 2x$$

$\Rightarrow \sin x = -2\sin x \cos x \Rightarrow \cos x = -\frac{1}{2} \Rightarrow x = \frac{2\pi}{3}$

So, $\left(\frac{2\pi}{3}, \frac{2\pi}{3}\right)$

$f_{xx} = -\cos x - \cos(x+y)$ $f_{xx}\left(\frac{2\pi}{3}, \frac{2\pi}{3}\right) = +\frac{1}{2} - \left(-\frac{1}{2}\right) = +1$

$f_{yy} = -\cos y - \cos(x+y)$ $f_{yy}\left(\frac{2\pi}{3}, \frac{2\pi}{3}\right) = +\frac{1}{2} - \left(-\frac{1}{2}\right) = +1$

$f_{xy} = -\cos(x+y)$

$f_{xy} = -\frac{1}{2}$

$\Rightarrow D = (+1)(+1) - \left(-\frac{1}{2}\right)^2 = \frac{3}{4} > 0$, $\cancel{f_{xx} < 0}$ $f_{xx} > 0$

So, $\left(\frac{2\pi}{3}, \frac{2\pi}{3}\right)$ is a minimum.

Answer: $\left(\frac{2\pi}{3}, \frac{2\pi}{3}\right)$ is a minimum.

2. *(10 points)* Express the number 21 as a sum of three positive numbers such that the product of these three numbers is a maximum.

$x+y+z=21$

$f(x,y,z)=xyz$

$\Rightarrow f(x,y)=xy(21-x-y) = 21xy-x^2y-xy^2$

We want to maximize $f(x,y)$ subject to $x,y>0$.

$\nabla f(x,y) = \langle 21y-2xy-y^2,\ 21x-x^2-2xy\rangle$

$21y-2xy-y^2 = y(21-2x-y)=0=21x-x^2-2xy=x(21-x-2y)$

So, $21-2x-y=0 \Rightarrow y=21-2x$ and

$21-x-2y = 21-x-2(21-2x) = -21+3x=0 \Rightarrow \boxed{x=7}$

therefore $y=21-2(7)=7$ and $z=21-7-7=7$.

So, $(7,7,7)$ is a stationary point.

$f_{xx}=-2y$ $\quad f_{xx}(7,7)=-14$ $\quad D=(-14)^2-(-7)^2>0$

$f_{yy}=-2x$ $\quad f_{yy}(7,7)=-14$ $\quad f_{xx}<0$ so it's a max.

$f_{xy}=21-2x-2y$ $\quad f_{xy}(7,7)=-7$

Answer: Max is $(7,7,7)$ so $7^3=343$

3. *(10 points)* Find all critical points of the function $f(x,y)=e^{x^2+y^2-4y}$.

$e^{x^2+y^2-4y}$ is differentiable everywhere and there is no boundary.

$\nabla f(x,y) = \langle 2xe^{x^2+y^2-4y}, (2y-4)e^{x^2+y^2-4y}\rangle$

$= \langle 0,0\rangle$

when $x=0$, $y=2$. So, $(0,2)$ is the only critical point.

Answer: $(0,2)$.