**Math2210 Quiz 7** (Sections 12.5-12.7) Summer, 2008 Dylan Zwick

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Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. *(8 points)* Find the directional derivative of $f(x,y)=3x^2-2xy+5y^2$ at $p=(3,1)$ in the direction of $a=2i-j$.

$\vec{a} = 2\hat{i} - \hat{j}$ $\nabla f = \langle 6x-2y, -2x+10y \rangle$

$\hat{a} = \dfrac{\langle 2,-1\rangle}{\sqrt{2^2+(-1)^2}} = \left\langle \dfrac{2}{\sqrt{5}}, -\dfrac{1}{\sqrt{5}} \right\rangle$ $\nabla f(3,1) = \langle 16, 4\rangle$

$$D_{\hat{a}}(f(3,1)) = \langle 16,4\rangle \cdot \left\langle \frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}} \right\rangle$$

$$= \frac{28}{\sqrt{5}} = \boxed{\frac{28\sqrt{5}}{5}}$$

Answer: $\dfrac{28\sqrt{5}}{5}$

2. *(8 points)* In what direction $u$ does $f(x,y)=4-x^2y^2+e^{2x}-3y$ increase most rapidly at $p=(0,3)$?

The gradient vector is in the direction of maximum increase.

$$\nabla f = \langle -2xy^2+2e^{2x}, -2x^2y-3\rangle$$

$\nabla f(0,3) = \langle 2,-3\rangle$ or any other vector in this direction.

Answer: $\langle 2,-3\rangle$

3. *(8 points)* Find $\dfrac{\partial z}{\partial x}$ given $F(x,y,z)=2x^2z-y^4-xyz^2=0$.

$$\frac{\partial F}{\partial x} = \frac{\partial F}{\partial x}\frac{\partial x}{\partial x} + \frac{\partial F}{\partial y}\frac{\partial y}{\partial x} + \frac{\partial F}{\partial z}\frac{\partial z}{\partial x} = 0 \qquad \frac{\partial y}{\partial x}=0,\ \frac{\partial x}{\partial x}=1$$

$$\Rightarrow \frac{\partial z}{\partial x} = \frac{-\partial F/\partial x}{\partial F/\partial z} = -\frac{4xz-yz^2}{2x^2-2xyz} = \frac{yz^2-4xz}{2x^2-2xyz}$$

Answer: $\boxed{\dfrac{yz^2-4xz}{2x^2-2xyz}}$

4. *(8 points)* Find $\frac{\partial w}{\partial t}$ for $w=e^{xy+z}$ given $x=s+t$ , $y=s-t$ and $z=t^2$ .

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial t}$$

$$= (ye^{xy+z})(1) + (xe^{xy+z})(-1) + (e^{xy+z})(2t)$$

$$= (y-x+2t)e^{xy+z}$$

$$y-x = (s-t)-(s+t) = -2t$$

$$\Rightarrow (-2t+2t)e^{xy+z} = 0\cdot e^{xy+z} = \boxed{0}$$

Answer: $0$

5. *(8 points)* Find the equation of the tangent plane to $3x^2+y^2+3z^2=9$ at $(1,0,\sqrt{2})$ .

$$F(x,y,z) = 3x^2+y^2+3z^2-9$$

$$\nabla F(x,y,z) = \langle 6x, 2y, 6z \rangle$$

$$\nabla F(1,0,\sqrt{2}) = \langle 6, 0, 6\sqrt{2} \rangle$$

$$F(1,0,\sqrt{2}) = 9$$

The plane will be:

$$\langle 6, 0, 6\sqrt{2} \rangle \cdot \langle x-1, y, z-\sqrt{2} \rangle = 0$$

$$\Rightarrow 6x-6+6\sqrt{2}z-12 = 0$$

$$\Rightarrow 6x+6\sqrt{2}z = 18$$

$$\Rightarrow \boxed{x+\sqrt{2}z = 3}$$

Answer: $x+\sqrt{2}z=3$