Math2210 Quiz 6 (Sections 12.2-12.4) Summer, 2008 Dylan Zwick

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Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. *(10 points)* Find the slope of the tangent to the curve of intersection of the surface $z=\frac{1}{2}\sqrt{9x^2+9y^2-36}$ and the plane $y=1$ at the point $(2,1,\frac{3}{2})$.

y is constant, so take

$$\frac{\partial z}{\partial x} = \frac{1}{2}\left(\frac{\frac{1}{2}}{\sqrt{9x^2+9y^2-36}}\right)(18x) = \frac{9x}{2\sqrt{9x^2+9y^2-36}}$$

$$\frac{\partial z}{\partial x}(2,1,\tfrac{3}{2}) = \frac{9(2)}{2\sqrt{9(2^2)+9(1^2)-36}} = \frac{18}{6} = \boxed{3}$$

slope = 3

2. *(5 points each)* Find the limit (or show that it does not exist).

(a) $\lim\limits_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{-3x^2-3y^2}$.

Switch to $r=\sqrt{x^2+y^2}$

$$\Rightarrow \lim_{r\to 0}\frac{\sin(r^2)}{-3(r^2)} \overset{\text{L'Hospitals}}{=} \lim_{r\to 0}\frac{2r\cos(r^2)}{-6r} = -\lim_{r\to 0}\frac{\cos(r^2)}{3} = \boxed{-\frac{1}{3}}$$

Answer : $-\frac{1}{3}$

(b) $\lim\limits_{(x,y)\to(0,0)} \frac{4xy}{x^2+y^2}$

For the line $x=0$ we get $\lim\limits_{y\to 0}\frac{4(0)y}{0^2+y^2} = \lim\limits_{y\to 0} 0 = 0.$

For the line $x=y$ we get $\lim\limits_{x\to 0}\frac{4x^2}{x^2+x^2} = \lim\limits_{x\to 0} 2 = 2.$

$2\neq 0$ So, the limit is not defined.

Answer: Does not exist.

3. *(10 points)* Find the gradient of f, ∇f, for $f(x,y)=\dfrac{-4x^3y-x}{y^2}$

$$\frac{\partial f}{\partial x} = \frac{-12x^2y-1}{y^2}$$

$$\frac{\partial f}{\partial y} = \frac{y^2(-4x^3)-(-4x^3y-x)(2y)}{(y^2)^2}$$

$$= \frac{-4x^3y^2+8x^3y^2+2xy}{y^4}$$

$$= \frac{4x^3y^2+2xy}{y^4} = \frac{4x^3y+2x}{y^3}$$

$$\Rightarrow \nabla f = \left\langle -\left(\frac{12x^2y+1}{y^2}\right), \frac{4x^3y+2x}{y^3} \right\rangle$$

∇f = $\left\langle -\left(\dfrac{12x^2y+1}{y^2}\right), \dfrac{4x^3y+2x}{y^3} \right\rangle$

4. *(10 points)* Find the equation of the tangent plane to $f(x,y)=y+2xy-6x^3y^2$ at $p_o=(1,1)$.

$$\nabla f(x,y) = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle = \langle 2y-18x^2y^2, 1+2x-12x^3y \rangle$$

$$T(x,y) = f(\vec{p}_0) + \nabla f(\vec{p}_0) \cdot \langle \vec{p} - \vec{p}_0 \rangle$$

$$f(1,1) = 1+2(1)(1) - 6(1^3)(1^2) = -3$$

$$\nabla f(1,1) = \langle -16, -9 \rangle$$

$$T(x,y) = -3 + \langle -16,-9 \rangle \cdot \langle x-1, y-1 \rangle$$

$$= -3 - 16x + 16 - 9y + 9$$

So,

$$z = -16x - 9y + 22$$

or

$$\boxed{16x+9y+z=22}$$

Tangent Plane: $16x+9y+z=22$