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Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. Convert $(-2, \frac{\pi}{4}, 2)$ from cylindrical to spherical coordinates.

$$\rho = \sqrt{(-2)^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$$

$$\theta = \frac{\pi}{4}$$

$$\phi = \cos^{-1}\left(\frac{2}{2\sqrt{2}}\right) = \frac{\pi}{4}$$

Answer: $(2\sqrt{2}, \frac{\pi}{4}, \frac{\pi}{4})$

2. Convert $r^2 \cos(2\theta) = z$ to Cartesian coordinates.

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$\Rightarrow r^2 \cos(2\theta) = r^2 \cos^2\theta - r^2 \sin^2\theta$$

$$= (r \cos\theta)^2 - (r \sin\theta)^2$$

$$= x^2 - y^2$$

$$\Rightarrow x^2 - y^2 = z$$

Answer: $x^2 - y^2 = z$

3. (5 points each) Let $f(x, y) = \frac{3x^2y - 2\sqrt{y-1}}{xy}$.

(a) Find $f(2, 1) = \underline{6}$

$$f(2, 1) = \frac{3(2^2)(1) - 2\sqrt{1-1}}{(2)(1)} = 6$$

(b) Find $f\left(\frac{1}{t}, t^2\right) = \underline{\frac{3 - 2\sqrt{t^2-1}}{t}}$

$$\begin{aligned} f\left(\frac{1}{t}, t^2\right) &= \frac{3\left(\frac{1}{t}\right)^2 t^2 - 2\sqrt{t^2-1}}{\left(\frac{1}{t}\right)t^2} \\ &= \frac{3 - 2\sqrt{t^2-1}}{t} \end{aligned}$$

(c) What is the domain? Domain: ~~any~~ $y \geq 1$ $x \neq 0$

(d) Find $f_x(2, 1) = \underline{3}$

$$f_x(x, y) = 3 + \frac{2\sqrt{y-1}}{x^2y}$$

$$f_x(2, 1) = 3 + \frac{2\sqrt{1-1}}{(2^2)(1)} = 3$$