Math2210 Quiz 4 (Sections 14.4, 14.5, 14.6, 14.7, 14.8, 14.9) Summer, 2008

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Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. Find a parametric equation for the line perpendicular to both of the vectors $\boldsymbol{a}=4\boldsymbol{i}-2\boldsymbol{j}+1\boldsymbol{k}$ and $\boldsymbol{b}=-2\boldsymbol{i}-\boldsymbol{k}$ and that passes through the origin (0,0,0).

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & -2 & 1\\ 2 & 0 & -1\end{vmatrix}=2\hat{i}+6\hat{j}+4\hat{k}=\langle a,b,c\rangle$$

$$(x_0,y_0,z_0)=(0,0,0)$$

$$x(t)=x_0+at=2t$$
$$y(t)=y_0+bt=6t$$
$$z(t)=z_0+ct=4t$$

Answer 1: $x(t)=2t \quad y(t)=6t \quad z(t)=4t$

2. Find the parametric equations of the line through (4, 1, 3) and (6, -1, 2).

$$\vec{v}=(6,-1,2)-(4,1,3)=(2,-2,-1)=\langle a,b,c\rangle$$
$$(x_0,y_0,z_0)=(4,1,3)$$

$$\Rightarrow x(t)=4+2t$$
$$y(t)=1-2t$$
$$z(t)=\cancel{3+t}\ 3-t$$

Answer 2: $x(t)=4+2t \quad y(t)=1-2t \quad z(t)=3-t$

3. Name the type of quadric surface given by $4x^2 + 25y^2 - 100z = 0$.

Type of surface: Elliptic Paraboloid

4. Change $(5, \frac{\pi}{3}, -1)$ from cylindrical coordinates to Cartesian.

$$x = r\cos\theta = 5\cos\left(\frac{\pi}{3}\right) = \frac{5}{2}$$
$$y = r\sin\theta = 5\sin\left(\frac{\pi}{3}\right) = \frac{5\sqrt{3}}{2}$$
$$z = -1$$

Answer: $\left(\frac{5}{2}, \frac{5\sqrt{3}}{2}, -1\right)$

Extra Credit: (5 pts) Change $(2\sqrt{3}, 6, -4)$ from Cartesian coordinates to spherical.

$$\rho = \sqrt{(2\sqrt{3})^2 + 6^2 + (-4)^2} = \sqrt{12 + 36 + 16} = \sqrt{64} = 8$$
$$\theta = \tan^{-1}\left(\frac{6}{2\sqrt{3}}\right) = \tan^{-1}\left(\sqrt{3}\right) = \frac{\pi}{3}$$
$$\phi = \cos^{-1}\left(\frac{-4}{8}\right) = \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$$

Extra Credit Answer: $\left(8, \frac{\pi}{3}, \frac{2\pi}{3}\right)$