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Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. If  $\vec{a} = \langle 3, 3, 1 \rangle$ ,  $\vec{b} = \langle -2, -1, 0 \rangle$  and  $\vec{c} = \langle -2, -3, -1 \rangle$ ,

(a) find  $\vec{a} \times (\vec{b} + \vec{c})$ .

$$\vec{b} + \vec{c} = \langle -2 + (-2), -1 + (-3), 0 + (-1) \rangle = \langle -4, -4, -1 \rangle$$

$$\begin{aligned} \vec{a} \times (\vec{b} + \vec{c}) &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 3 & 1 \\ -4 & -4 & -1 \end{vmatrix} = (-3 - (-4))\hat{i} + (-4 - (-3))\hat{j} + (-12 - (-12))\hat{k} \\ &= \hat{i} - \hat{j} = \boxed{\langle 1, -1, 0 \rangle} \end{aligned}$$

$$\vec{a} \times (\vec{b} + \vec{c}) = \underline{\langle 1, -1, 0 \rangle}$$

(b) find  $\vec{a} \cdot (\vec{b} \times \vec{c})$

$$\begin{aligned} \vec{b} \times \vec{c} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & -1 & 0 \\ -2 & -3 & -1 \end{vmatrix} = (1 - 0)\hat{i} + (0 - 2)\hat{j} + (6 - 2)\hat{k} \\ &= \hat{i} - 2\hat{j} + 4\hat{k} = \langle 1, -2, 4 \rangle \end{aligned}$$

$$\begin{aligned} \vec{a} \cdot (\vec{b} \times \vec{c}) &= \langle 3, 3, 1 \rangle \cdot \langle 1, -2, 4 \rangle \\ &= 3 - 6 + 4 = \boxed{1} \end{aligned}$$

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \underline{1}$$

2. Find the equation of the plane passing through  $P(4, -2, 1)$  and perpendicular to  $n=2i-5j+3k$ .

$$\vec{n} = \langle A, B, C \rangle = \langle 2, -5, 3 \rangle$$

$$(x_0, y_0, z_0) = P = (4, -2, 1)$$

So, the plane is:

$$2(x-4) - 5(y+2) + 3(z-1) = 0$$

$$\Rightarrow 2x - 8 - 5y - 10 + 3z - 3 = 0$$

$$\Rightarrow \boxed{2x - 5y + 3z = 21}$$

Plane:  $2x - 5y + 3z = 21$

3. Find the distance between the parallel planes  $5x - 3y - 2z = 5$  and  $-5x + 3y + 2z = 7$ .

$$L = \frac{|Ax_0 + By_0 + Cz_0 - D|}{\sqrt{A^2 + B^2 + C^2}}$$

where  $\langle A, B, C \rangle$  is a normal vector to  $-5x + 3y + 2z = 7$  given by  $\langle -5, 3, 2 \rangle$  and  $D=7$ .  $(x_0, y_0, z_0)$  is a point on  $5x - 3y - 2z = 5$ . Pick point  $(1, 0, 0)$  and get

$$L = \frac{|-5(1) + 3(0) + 2(0) - 7|}{\sqrt{25 + 9 + 4}} = \boxed{\frac{12}{\sqrt{38}}}$$

distance:  $\frac{12}{\sqrt{38}}$