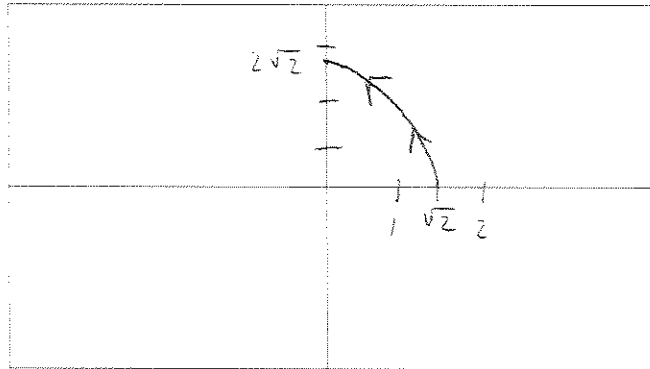


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Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. For  $x = \sqrt{3-t}$  and  $y = 2\sqrt{t-1}$  such that  $1 \leq t \leq 3$ , eliminate the parameter and graph the curve. Indicate if the curve is simple and/or closed.



$$x^2 = 3 - t$$

$$\Rightarrow t = 3 - x^2$$

$$y^2 = 4(t-1)$$

$$\Rightarrow t = \frac{y^2}{4} + 1$$

$$3 - x^2 = \frac{y^2}{4} + 1$$

$$\Rightarrow \boxed{x^2 + \frac{y^2}{4} = 2}$$

from  $t = 1$

$$x = \sqrt{2}$$

$$y = 0$$

$t = 3$

$$x = 0$$

$$y = 2\sqrt{2}$$

Simple:  T or  F (circle one)Closed:  T or  F (circle one)

2. Find the distance between the points  $(-2, 1, 2)$  and  $(4, -3, 6)$ .

$$D = \sqrt{(4 - (-2))^2 + (-3 - 1)^2 + (6 - 2)^2}$$

$$= \sqrt{36 + 16 + 16}$$

$$= \sqrt{68} = 2\sqrt{17}$$

distance =  $2\sqrt{17}$

3. Find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  (without eliminating the parameter) for  $x=4t^5$  and  $y=-2t^3$  such that  $t \neq 0$ .

$$\frac{dy}{dt} = -6t^2 \quad \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-6t^2}{20t^4} = -\frac{3}{10t^2}$$

$$\frac{dx}{dt} = 20t^4 \quad \frac{d}{dt} \left( \frac{dy}{dx} \right) = \frac{3}{5t^3}$$

$$\frac{d^2y}{dx^2} = \frac{3/5t^3}{20t^4} = \frac{3}{100t^7}$$

$$\frac{dy}{dx} = \frac{-\frac{3}{10t^2}}{\frac{3}{100t^7}}$$

4. Find the equation of the sphere that has the line segment joining the two points in question #2 as a diameter.

The center is the midpoint:

$$\left( \frac{4+(-2)}{2}, \frac{-3+1}{2}, \frac{6+2}{2} \right) = (1, -1, 4)$$

The radius is  $\frac{1}{2}$  the diameter

$$r = \frac{2\sqrt{17}}{2} = \sqrt{17}$$

So, the sphere's equation is:

$$(x-1)^2 + (y+1)^2 + (z-4)^2 = (\sqrt{17})^2 = 17$$

Center of sphere: (1, -1, 4)

Equation of sphere:  $(x-1)^2 + (y+1)^2 + (z-4)^2 = 17$