Math2210 Quiz 12 (Sections 14.2-14.3) Summer, 2008 Dylan Zwick

Name Dylan Zwick Date 07/28/08

Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. (10 points) Determine whether $\boldsymbol{F}(x, y)=(-e^{-x}\ln y)\boldsymbol{i}+(e^{-x}y^{-1})\boldsymbol{j}$ is conservative. If so, find f such that $\boldsymbol{F}=\nabla f$. If not, state that $\boldsymbol{F}$ is not conservative.

$M(x,y) = -e^{-x}\ln y \qquad N(x,y) = e^{-x}y^{-1}$

$\frac{\partial M}{\partial y} = -\frac{e^{-x}}{y} \qquad \frac{\partial N}{\partial x} = -\frac{e^{-x}}{y}$

So, conservative.

$\frac{\partial f}{\partial x} = -e^{-x}\ln y \Rightarrow f(x,y) = e^{-x}\ln y + C(y)$

$\Rightarrow \frac{\partial f}{\partial y} = \frac{e^{-x}}{y} + C'(y) = \frac{e^{-x}}{y}$

So, $C'(y) = 0$ and so $C(y) = c$

Conservative: True or False (circle one)

If conservative, f = $e^{-x}\ln y + c$

2. *(10 points)* Show that the line integral is independent of path and then evaluate it.

$$\int_{(0,0,0)}^{(\pi,\pi,0)} (\cos x+2yz)\,dx+(\sin y+2xz)\,dy+(z+2xy)\,dz$$

Prove independence of path:

$$\vec{F} = \langle \cos x + 2yz,\ \sin y + 2xz,\ z+2xy \rangle$$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \cos x+2yz & \sin y+2xz & \cancel{z+2xy} \end{vmatrix} = (2x-2x)\hat{i} + (2y-2y)\hat{j} + (2z-2z)\hat{k}$$

$$= \vec{0}$$

Evaluate integral: Answer $\boxed{2}$

Take the parametrization:

$x(t) = \pi t \qquad dx = \pi\,dt$

$y(t) = \pi t \qquad dy = \pi\,dt$

$z(t) = 0 \qquad dz = 0$

$0 \le t \le 1$

$$\Rightarrow \int_0^1 \cos(\pi t)(\pi\,dt) + \sin(\pi t)(\pi\,dt)$$

$$= \pi \int_0^1 [\cos(\pi t) + \sin(\pi t)]\,dt$$

$$= \pi \left[\frac{\sin(\pi t)}{\pi} - \frac{\cos(\pi t)}{\pi} \right] \Bigg|_0^1$$

$$= \pi \left(\frac{1}{\pi} - \left(-\frac{1}{\pi}\right)\right) = \boxed{2}$$

3. *(10 points)* Evaluate the line integral $\int_C xz\,dx + (y+z)\,dy + x\,dz$ where C is the curve $x=e^t$, $y=e^{-t}$, $z=e^{2t}$ and t goes from 0 to 1.

$dx = e^t dt$

$dy = -e^{-t} dt$

$dz = 2e^{2t}$

$$\int_0^1 \left[e^{3t}(e^t dt) + (e^{-t} + e^{2t})(-e^{-t} dt) + e^t(2e^{2t}) \right] dt$$

$$= \int_0^1 \left[e^{4t} - e^{-2t} - e^t + 2e^{3t} \right] dt$$

$$= \frac{e^{4t}}{4} + \frac{e^{-2t}}{2} - e^t + \frac{2}{3}e^{3t} \Big|_0^1$$

$$= \left(\frac{e^4}{4} + \frac{e^{-2}}{2} - e + \frac{2}{3}e^3 \right) - \left(\frac{1}{4} + \frac{1}{2} - 1 + \frac{2}{3} \right)$$

$$= \frac{e^4}{4} + \frac{e^{-2}}{2} - e + \frac{2}{3}e^3 - \frac{5}{12}$$

Answer: $\frac{e^4}{4} + \frac{e^{-2}}{2} - e + \frac{2}{3}e^3 - \frac{5}{12}$

4. *(10 points)* (True or False)

T or F (circle one) There are 8 possible orders of integration for a triple integral.

T or F (circle one) The cross product of two unit vectors is another unit vector.

T or F (circle one) 2+2 = 4