Math2210 Quiz 11 (Sections 13.7, 13.8, 14.1) Summer, 2008 Dylan Zwick

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Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. *(10 points each)* Set up the triple integral (do not evaluate!!!) to find the volume of the solid inside the sphere $x^2+y^2+z^2=16$, outside the cone $z=\sqrt{x^2+y^2}$, and above the xy-plane. (Use **spherical coordinates**)

$$V = \int_{\pi/4}^{\pi/2}\int_0^{2\pi}\int_0^4 \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$$

$$= \int_{\pi/4}^{\pi/2}\int_0^{2\pi} \frac{\rho^3}{3}\sin\phi \Big|_0^4 \, d\theta \, d\phi$$

$$= \int_{\pi/4}^{\pi/2}\int_0^{2\pi} \frac{64}{3}\sin\phi \, d\theta \, d\phi = \int_{\pi/4}^{\pi/2} \frac{128}{3}\pi \sin\phi \, d\phi$$

$$= -\frac{128}{3}\pi\cos\phi \Big|_{\pi/4}^{\pi/2} = 0 - \left(-\frac{128}{3}\pi\frac{\sqrt{2}}{2}\right)$$

$$= \frac{64\sqrt{2}}{3}\pi$$

Answer: $\boxed{\frac{64\sqrt{2}}{3}\pi}$

2. *(10 points)* Find $\nabla\cdot\boldsymbol{F}$ (div $\boldsymbol{F}$) and $\nabla\times\boldsymbol{F}$ (curl $\boldsymbol{F}$) given $\boldsymbol{F}=3xy\,\boldsymbol{i}-2x^2\,\boldsymbol{j}+yz^3\,\boldsymbol{k}$.

$$\nabla\vec{F} = 3y + 0 + 3yz^2 = 3y+3yz^2 = 3y(1+z^2)$$

$$\nabla\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3xy & -2x^2 & yz^3 \end{vmatrix} = z^3\hat{i} + 0\hat{j} + (-4x-3x)\hat{k}$$

$$= z^3\hat{i} - 7x\hat{k}$$

$\nabla\cdot\boldsymbol{F}$ = $3y(1+z^2)$

$\nabla\times\boldsymbol{F}$ = $z^3\hat{i} - 7x\hat{k}$

3. *(10 points)* Evaluate $\int_1^2 \int_1^z \int_0^{\sqrt{\frac{x}{z}}} 12xyz \; dy\,dx\,dz$.

$$\int_1^2 \int_1^z \int_0^{\sqrt{\frac{y}{z}}} 12xyz \, dy \, dx \, dz$$

$$= \int_1^2 \int_1^z 6xy^2z \Big|_0^{\sqrt{\frac{x}{z}}} dx \, dz$$

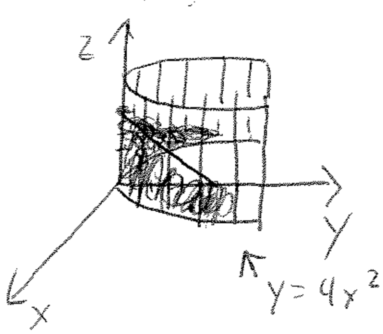
$$= \int_1^2 \int_1^z 6x^2 \, dx \, dz$$

$$= \int_1^2 \left(2x^3 \Big|_1^z\right) dz = \int_1^2 (2z^3 - 2) \, dz = \frac{z^4}{2} - 2z \Big|_1^2$$

$$= (8-4) - \left(\tfrac{1}{2} - 2\right) = 11/2$$

Answer: $\boxed{11/2}$

4. *(10 points)* Set up the triple integral **(do not evaluate!!!)** to find the volume of the solid in the first octant bounded by $y = 4x^2$ and $3y + 4z = 12$.



$0 \le y \le 4$

So, we have $-1 \le x \le 1$

and $4x^2 \le y \le 4$

and $0 \le z \le 3 - \frac{3}{4}y$

$$\int_{-1}^{1} \int_{4x^2}^{4} \int_0^{3-\frac{3}{4}y} 1 \, dz \, dy \, dx$$

Answer: $\boxed{\int_{-1}^{1} \int_{4x^2}^{4} \int_0^{3-\frac{3}{4}y} dz \, dy \, dx}$