Math2210 Quiz 10 (Sections 13.3, 13.4, 13.6) Summer, 2008 Dylan Zwick

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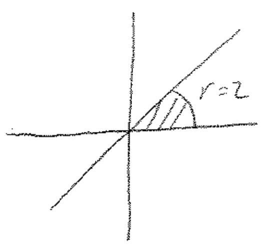
Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. *(10 points)* Evaluate $\iint_S \sqrt{4-x^2-y^2}\,dA$ using polar coordinates, where S is the first quadrant sector of the circle $x^2+y^2=4$ between $y=0$ and $y=x$.



$$\int_0^{\pi/4}\int_0^2 \sqrt{4-r^2}\, r\,dr\,d\theta$$

$u = 4-r^2$

$du = -2r\,dr$

$\Rightarrow -\frac{du}{2} = r\,dr$

$$\Rightarrow \int_0^{\pi/4}\int_4^0 -\frac{\sqrt{u}}{2}\,du\,d\theta$$

$$= \int_0^{\pi/4}\int_0^4 \frac{\sqrt{u}}{2}\,du\,d\theta$$

$$= \int_0^{\pi/4}\left[\frac{u^{3/2}}{3}\Big|_0^4\right]d\theta$$

$$= \int_0^{\pi/4} \frac{8}{3}\,d\theta = \frac{8}{3}\theta\Big|_0^{\pi/4} = \frac{8}{3}\left(\frac{\pi}{4}\right) = \frac{2}{3}\pi$$

Answer: $\boxed{\frac{2}{3}\pi}$

2. *(10 points)* Find the area of the surface $z=\sqrt{9-y^2}$ that is directly above the square with vertices (1,0), (3,0), (3,2) and (1,2).

$$\frac{\partial z}{\partial x} = 0 \qquad \frac{\partial z}{\partial y} = \frac{-y}{\sqrt{9-y^2}}$$

$$SA = \int_1^3\int_0^2 \sqrt{1+\left(\frac{-y}{\sqrt{9-y^2}}\right)^2+0^2}\,dy\,dx$$

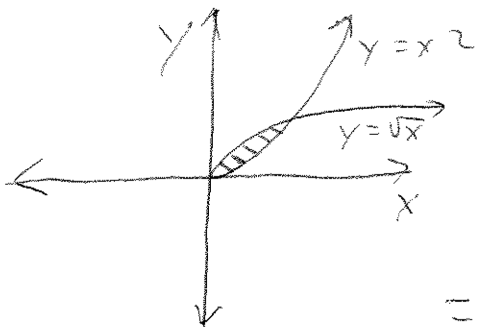
$$= \int_1^3\int_0^2 \sqrt{1+\frac{y^2}{9-y^2}}\,dy\,dx$$

$$= \int_1^3\int_0^2 \frac{3}{\sqrt{9-y^2}}\,dy\,dx = \int_1^3 3\sin^{-1}\left(\frac{y}{3}\right)\Big|_0^2\,dx$$

$$= \int_1^3 3\sin^{-1}\left(\frac{2}{3}\right)dx = 3\sin^{-1}\left(\frac{2}{3}\right)x\Big|_1^3 = 6\sin^{-1}\left(\frac{2}{3}\right)$$

Answer: $\boxed{6\sin^{-1}\left(\frac{2}{3}\right)}$

3. *(10 points)* $\iint_S (x+y)\,dA$ where S is the region between $y=x^2$ and $y=\sqrt{x}$.



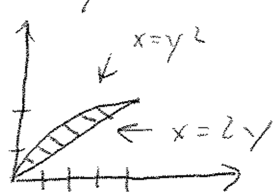
$$0 \le x \le 1$$
$$x^2 \le y \le \sqrt{x}$$

$$\int_0^1 \int_{x^2}^{\sqrt{x}} (x+y)\,dy\,dx$$
$$= \int_0^1 \left(xy + \frac{y^2}{2} \Big|_{x^2}^{\sqrt{x}} \right) dx$$
$$= \int_0^1 \left[\left(x^{3/2} + \frac{x}{2} \right) - \left(x^3 + \frac{x^4}{2} \right) \right] dx$$
$$= \frac{2}{5} x^{5/2} + \frac{x^2}{4} - \frac{x^4}{4} - \frac{x^5}{10} \Big|_0^1$$
$$= \frac{2}{5} + \frac{1}{4} - \frac{1}{4} - \frac{1}{10} = \frac{3}{10}$$

Answer: $\boxed{3/10}$

4. *(10 points)* Rewrite $\int_0^2 \int_{y^2}^{2y} f(x,y)\,dx\,dy$ as an iterated integral with the order of integration switched.

The region is



Switching the variables we get:

$$0 \le x \le 4$$
$$\frac{x}{2} \le y \le \sqrt{x} \quad \Rightarrow \quad \int_0^4 \int_{\frac{x}{2}}^{\sqrt{x}} f(x,y)\,dy\,dx$$

Answer: $\int_0^4 \int_{x/2}^{\sqrt{x}} f(x,y)\,dy\,dx$