Math2210 Midterm 2 Summer, 2008 Dylan Zwick

Name _____ Date _____

Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. (10 pts) Find the directional derivative of $f(x,y,z)=3xy+2y^2z^2-x^3$ at $\boldsymbol{p}=(4,1,0)$ in the direction of $\boldsymbol{a}=-2\boldsymbol{i}+\boldsymbol{j}+2\boldsymbol{k}$.

Answer: _____

2. (15 pts) Find a point on the surface $F(x, y, z)=2x^2+3y^2-z=0$ where the tangent plane is parallel to the plane $8x-3y-z=0$.

Answer : _____

3. (10 pts) Name and sketch the graph (in 3d) for $x^2+4z^2-9=16y^2$.

Name of graph: _____

Along which axis: _____

4. (10 pts) Describe the largest set S on which $f(x, y, z)=\ln(4-x^2-y^2-z^2)$ is continuous.

Answer : _____

5. (15 pts) Find all critical points for $f(x,y)=x^3+y^3-2xy+4$. Determine whether each point is a minimum, maximum or saddle point.

Critical point(s) (Specify whether they're min, max or saddle.):

6. For $z = f(x, y) = -2x^3 y^2 + 5\ln(xy)$, find

(a) (10 pts) $\frac{\partial z}{\partial y}$ at (2, -1)

Answer : _____

(b) (10 pts) f_{xy}

Answer : _____

7. (a) (10 pts) Convert $2x^2+2y^2=5y+81$ from a Cartesian coordinate equation into an equation in cylindrical coordinates.

Answer : _____

(b) (10 pts) Convert $\rho=-3\sec\phi$ from a spherical coordinate equation into an equation in Cartesian coordinates.

Answer : _____

8. For $f(x, y) = x^2 \sin y + 3xy - 5$,

(a) (10 pts) Find ∇f .

Answer : _____

(b) (10 pts) Find the equation of the tangent plane at (-2, 0).

Answer: _____

9. (10 pts) Use the total differential dz to approximate the change in z as (x, y) moves from P(-2, -0.5) to Q(-2.03, -0.51) for $z = \arctan(xy)$

Answer : _____

10.(10 pts) Find $\frac{\partial w}{\partial u}$ if $w(x, y) = x^3 - xy^2 + 2y$, $x = \sqrt{u - v}$, $y = 3uv$. (Your answer must be only in terms of u and v. Don't bother simplifying your answer.)

Answer : _____

11. (10 pts each) Find the limit, if it exists. (Show all your reasoning.)

(a) $\lim_{(x,y)\to(1,2)} \frac{8x^3 - y^3}{x^3 + 8y^3}$

Answer : _____

(b) $\lim_{(x,y)\to(0,0)} \frac{x - 5y}{x + 2y}$

Answer : _____

Extra Credit: (8 pts) Decide whether each of the following statements is either true or false.

1. $\vec{u}+(\vec{v}\times\vec{w})$ makes sense.

True or False (circle one)

2. The gradient of f is perpendicular to the graph of $z=f(x,y)$.

True or False (circle one)

3. If f is continuous at (x, y), then it's differentiable there.

True or False (circle one)

4. $(\vec{a}\cdot\vec{b})+\vec{c}$ makes sense.

True or False (circle one)

5. If $\vec{u}\cdot\vec{v}=0$ and $\vec{u}\times\vec{v}=0$, then $\vec{u}$ or $\vec{v}$ is the zero vector.

True or False (circle one)

6. If $\lim_{y\to 0} f(y,y)=L$, then $\lim_{(x,y)\to(0,0)} f(x,y)=L$.

True or False (circle one)

7. $\dfrac{\partial^3 f}{\partial x\,\partial y^2}=f_{yxy}$ always.

True or False (circle one)

8. The level curves for $5z=\sqrt{25-4x^2-y^2}$ are ellipses.

True or False (circle one)