Math2210 Midterm 1 Summer, 2008 Dylan Zwick

Name _____ Date _____

Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. For $x = 2t^2 + 1$ and $y = 4t - 5$ such that $-1 \le t \le 1$, do the following:

(a) (10 pts) Eliminate the parameter to obtain the corresponding Cartesian equation.

Answer 1(a): _____

(b) (10 pts) Graph the curve.

(c) (5 pts) Indicate if the curve is simple and/or closed.

Simple: T or F (circle one)

Closed: T or F (circle one)

2. (10 pts) Find the length of the curve given by $x=\cos t$ and $y=\ln(sec\,t+\tan t)-\sin t$ for $0\le t\le\frac{\pi}{4}$.

Answer 2: _____

3. (15 pts) For position vector given by $\boldsymbol{r}(t)=e^{t}\boldsymbol{i}+e^{-t}\boldsymbol{j}+2t\boldsymbol{k}$, find the velocity and acceleration vectors and the speed at $t=\ln 2$.

$\boldsymbol{v}(t)$ = _____

$\boldsymbol{a}(t)$ = _____

speed at $t=\ln 2$ = _____

4. (10 pts) Find the limit, if it exists. $\lim_{t\to 0}\left[\frac{2t\sin t}{t^2}\boldsymbol{i}-\frac{4t^3}{t^2-t}\boldsymbol{j}+\frac{\tan t}{\sin t}\boldsymbol{k}\right]$

Answer (4) : _____

5. (10 pts) Find the equation of the sphere that has the line segment joining (-1, 4, 3) and (3, 0, 5) as a diameter.

Radius = _____ units

center = _____

Eqn of sphere: _____

6. (10 pts each) Let $\boldsymbol{a}=\langle 2,-1,1\rangle$, $\boldsymbol{b}=\langle -3,-1,4\rangle$ and $\boldsymbol{c}=5\boldsymbol{i}+2\boldsymbol{j}$. Find each of the following.

(a) $2\boldsymbol{a}-3\boldsymbol{c}$

$2\boldsymbol{a}-3\boldsymbol{c}$ = _____

(b) $\boldsymbol{a}\cdot(\boldsymbol{b}+\boldsymbol{c})$

$\boldsymbol{a}\cdot(\boldsymbol{b}+\boldsymbol{c})$ = _____

(c) $\boldsymbol{b}\cdot\boldsymbol{c}-|\boldsymbol{b}|$

$\boldsymbol{b}\cdot\boldsymbol{c}-|\boldsymbol{b}|$ = _____

(Note: This is # 6 continued.) $\boldsymbol{a}=\langle 2,-1,1\rangle$, $\boldsymbol{b}=\langle -3,-1,4\rangle$ and $\boldsymbol{c}=5\boldsymbol{i}+2\boldsymbol{j}$

(d) $\hat{\boldsymbol{c}}$ (the unit vector)

$\hat{\boldsymbol{c}}$ = _____

(e) $\boldsymbol{a}\times(\boldsymbol{b}\times\boldsymbol{c})$

$\boldsymbol{a}\times(\boldsymbol{b}\times\boldsymbol{c})$ = _____

(f) $\boldsymbol{a}\cdot(\boldsymbol{b}\times\boldsymbol{c})$

$\boldsymbol{a}\cdot(\boldsymbol{b}\times\boldsymbol{c})$ = _____

7. (10 pts each) For $\boldsymbol{a} = \boldsymbol{i} - 3\,\boldsymbol{j} + 4\,\boldsymbol{k}$ and $\boldsymbol{b} = 2\,\boldsymbol{i} - \boldsymbol{k}$, find each of the following:
(a) Direction cosines for $\boldsymbol{a}$.

$\cos\alpha$ = _____

$\cos\beta$ = _____

$\cos\gamma$ = _____

(b) The angle θ between $\boldsymbol{a}$ and $\boldsymbol{b}$. (Just write a simplified expression. If you don't have a calculator just write the numerical formula for the angle.)

θ = _____

(c) Find the projection of $\boldsymbol{b}$ onto $\boldsymbol{a}$.

Projection of $\boldsymbol{b}$ onto $\boldsymbol{a}$ = _____

8. (10 pts each) For the planes given by $3x + y + z = 7$ and $5x + 3z = 13$, answer the following questions.

(a) Find the line of intersection between the planes and write that line in parametric equations.

Line: _____

(b) Find the equation of the plane that is perpendicular to the line of intersection and goes through the point (2, 1, 3).

Equation of plane: _____

Extra Credit: (9 pts)

A luxury cruiseliner is traveling due west at only 8 miles per hour. A woman on the ship is running across the ship, heading due north, at 6 miles per hour. What are the magnitude and direction of her velocity relative to the surface of the water? (If you don't have a calculator, just give the angle in simplified form.)

velocity magnitude: _____

velocity direction: _____