Math2210 Final Exam Summer, 2008 Dylan Zwick

Name _____ Date _____

Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown. All answers should be completely simplified, unless otherwise stated.

1. *(15 points)* For position vector given by $\boldsymbol{r}(t)=(3t^3-5t)\boldsymbol{i}+(2t^2+7)\boldsymbol{j}$, find the velocity and acceleration vectors and the speed at $t=\frac{1}{2}$.

$\boldsymbol{v}(t)$ = _____

$\boldsymbol{a}(t)$ = _____

speed at $t=\frac{1}{2}$ = _____

2. *(20 points)* Let $\boldsymbol{a}=\langle 1,-3\rangle$, $\boldsymbol{b}=\langle 2,6\rangle$ and $\boldsymbol{c}=\langle -2,5\rangle$. Find each of the following.

(a) $2\boldsymbol{a}-3\boldsymbol{c}$

$2\boldsymbol{a}-3\boldsymbol{c}$ = _____

(b) $\boldsymbol{a}\cdot(\boldsymbol{b}+\boldsymbol{c})$

$\boldsymbol{a}\cdot(\boldsymbol{b}+\boldsymbol{c})$ = _____

(c) projection of $\boldsymbol{a}$ onto $\boldsymbol{b}$

projection of $\boldsymbol{a}$ onto $\boldsymbol{b}$ = _____

(d) $\hat{\boldsymbol{a}}$ (the unit vector)

$\hat{\boldsymbol{a}}$ = _____

3. For the points A(-1, 2, 3), B(4, 1, 5) and C(1, 1, -1)

(a) *(10 points)* Write the equation of the plane through points A, B and C.

plane normal vector = _____

Eqn of plane: _____

(b) *(10 points)* Write a set of parametric equations for the line through point B and perpendicular to the plane in part (a).

Line: _____

4. *(15 points)* Find the directional derivative of $f(x, y, z)=x^3 y-y^2 z^2$ at $\boldsymbol{p}=(-1,3,-2)$ in the direction of $\boldsymbol{a}=\boldsymbol{i}-2\boldsymbol{k}$.

Answer 4: _____

5. *(20 points)* For the surface $F(x, y, z) = 4x^2 - 3xy + 2y^2 + 4z - z^2 = 4$

(a) Find the equation of the tangent plane at the point (1, 1, 1).

Answer 5(a): _____

(b) Find a point on the surface where the tangent plane is parallel to the plane $14x - 11y + 4z = 19$.

Answer 5(b):_____

6. *(15 points)* Find all critical points of the function $f(x,y)=e^{x^2+y^2-4y}$.

Answer 6: _____

7. *(15 points)* For the solid inside $x^2+y^2=9$ bounded above by the sphere $x^2+y^2+z^2=16$ and below by the xy-plane, do the following.

(a) Set up the volume integral in Cartesian coordinates.

Answer 7(a): _____

(b) Rewrite the integral using cylindrical coordinates.

Answer 7(b): _____

(c) Evaluate the volume (using the integral of your choice).

Answer 7(c): _____

8. *(20 points)* Given $\boldsymbol{F}(x, y, z) = 5x^2 y z^2 \boldsymbol{i} - 2y x^3 \boldsymbol{j} + y^4 z^3 \boldsymbol{k}$, calculate the following.

(a) div $\boldsymbol{F}$

div $\boldsymbol{F}$ = _____

(b) curl $\boldsymbol{F}$

curl $\boldsymbol{F}$ = _____

(c) $\nabla(\nabla \cdot \boldsymbol{F})$

$\nabla(\nabla \cdot \boldsymbol{F})$ = _____

(d) $\nabla \cdot (\nabla \times \boldsymbol{F})$

$\nabla \cdot (\nabla \times \boldsymbol{F})$ = _____

9. *(15 points)* Evaluate the line integral $\int_C (x^2+y^2)\,ds$ given C is the path given by $x=e^t \sin t$, $y=e^t \cos t$ and $0 \le t \le 3$.

Answer 9: _____

10. *(15 points)* Determine whether
$\boldsymbol{F}(x, y, z)=(y^2\cos x+z^3)\boldsymbol{i}+(2y\sin x-4)\boldsymbol{j}+(3xz^2+2)\boldsymbol{k}$ is conservative. If so, find f such that $\boldsymbol{F}=\nabla f$. If not, state that $\boldsymbol{F}$ is not conservative.

Conservative: True or False (circle one)

If conservative, f = _____

11. *(15 points)* Use Green's Theorem to evaluate $\oint_C (2xy)\,dx+(e^x+x^2)\,dy$ where C is the boundary of the triangle with vertices (1, 0), (2, 0) and (2, 2) oriented counter-clockwise.

Answer 11: _____

12. *(15 points)* Evaluate the integral $\int_0^{\frac{\pi}{4}} \int_{\cos(4z)}^{0} \int_0^{4yz} \cos\left(\frac{x}{y}\right) dx\, dy\, dz$.

Answer 12: _____

Extra Credit:

(10 points) Prove the identity:

$$\int_0^x \int_0^v \int_0^u f(t)\,dt\,du\,dv = (1/2)\int_0^x (x-t)^2 f(t)\,dt$$

Extra Credit 2:

(5 points) What is 5+2?

Answer : _____