Math 2210 - Midterm

University of Utah

Summer 2007

Name: _____

50 points possible.

1. (10 points) *Partial Derivatives*

   A function of two variables that satisfies Laplace's equation:

   $$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$$

   is said to be harmonic. Show that the functions defined below are harmonic functions.

   (a) (3 points)

   $$f(x, y) = x^3 y - xy^3$$

(b) (3 points)

$$f(x, y) = e^x \sin y$$

(c) (4 points)

$$f(x, y) = \ln\left(4x^2 + 4y^2\right)$$

2. (10 points) *Limits*

Determine each of the following limits, or state it does not exist and give an explanation as to why.

(a) (3 points)

$$\lim_{(x,y)\to(1,1)} \left(x^2 + y^2 + \cos\left(xy - 1\right)\right)$$

(b) (3 points)

$$\lim_{(x,y)\to(0,0)} \frac{\tan\left((x^2 + y^2)^2\right)}{(x^2 + y^2)^3}$$

(c) (4 points)

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2 + y^2}$$

3. (10 points) *Gradients*

   Calculate the gradients of the following functions:

   (a) (2 points)

   $$f(x, y) = e^{-xy}$$

   (b) (2 points)

   $$f(x, y, z) = x^3 y - y^2 z^2$$

   (2 points)

   For the function of two variables above, calculate the directional derivative at the point $(1, -1)$ in the direction of $\mathbf{v} = -\mathbf{i} = \sqrt{3}\mathbf{j}$.

(2 points)

For the function of three variables above, calculate the directional derivative at the point $(-2, 1, 3)$ in the direction of $\mathbf{v} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$.

(2 points)

For the function of three variables above at the point $(-2, 1, 3)$ provide the maximum value of the directional derivative and provide a unit vector in this maximal direction.

4. (10 points) *Extrema and Tangent Planes*

For the surface defined by:

$$z = x^3 + y^3 - 6xy$$

(a) (6 points)

Determine all the critical points of the function, and determine if these points are local maxima, local minima, or saddle points.

(b) (4 points)

Calculate the equation for the tangent plane to the function at the point $(3, 3, 0)$.

Note - The equation for the plane should be of the form

$$Ax + By + Cz = D.$$

5. (5 points) *Chain Rule*

For the implicitly defined curve:

$$x^2 \cos y - y^2 \sin x = 0$$

calculate $\dfrac{dy}{dx}$.

6. (5 points) *Chapter 11*

Find the symmetric equations of the tangent line to the curve

$$\mathbf{r}(t) = (2t^2, 4t, t^3)$$

at $t = 1$.