Math 2210 - Final Exam

University of Utah

Summer 2007

Name: ______________________________

1. (10 points)
   For the vectors:

$$\mathbf{a} = -3\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$$
and
$$\mathbf{b} = -\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}.$$

   Calculate:

   (a) (2 points)
   $\mathbf{a} + \mathbf{b}$

   (b) (2 points)
   $\mathbf{a} - \mathbf{b}$

(c) (3 points)
$\mathbf{a} \cdot \mathbf{b}$

(d) (3 points)
$\mathbf{a} \times \mathbf{b}$

2. (10 points)

   For the function:

$$f(x,y,z) = x^3y + y^2z^2$$

   Calculate:

   (a) (4 points)
   The gradient $\bigtriangledown f(x,y,z)$ of the function $f(x,y,z)$.

   (b) (4 points)
   The directional derivative at the point $\mathbf{p} = (-2,1,3)$ in the direction of the vector $\mathbf{a} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$.

   (c) (2 points)
   The maximum value of the directional derivative at the point $\mathbf{p} = (-2,1,3)$.

3. (10 points)

Evaluate the double integral:

$$\int\int_S e^{x^2+y^2}\,dA$$

Where S is the region enclosed by $x^2 + y^2 = 4$. Make a sketch of this region before you calculate the integral.

4. (10 points)

   Evaluate the line integral:

$$\int_C xe^y ds$$

   Where C is the line segment from $(-1, 2)$ to $(1, 1)$.

   Note - As always, ds here means the differential with respect to arc length.

5. (10 points)

   Evaluate the line integral:

$$\oint_C (e^{3x} + 2y)dx + (x^2 + \sin y)dy$$

   Where C is the rectangle with vertices $(2, 1)$, $(6, 1)$, $(6, 4)$, and $(2, 4)$ traversed in the counter-clockwise direction.

   Hint - Use Green's theorem.

6. (10 points)

   For the function:

$$f(x, y) = e^{-(x^2+y^2-4y)}$$

   find all the critical points, and indicate whether each such point gives a local maximum, a local minimum, or a saddle point.

7. (10 points)

   Find the volume of the solid bounded by the cylinders $x^2 = y$ and $z^2 = y$ and the plane $y = 1$.

   Hint - Your x and z limits depend only on y.

8. (10 points)

For the vector field:

$$\mathbf{F} = (6xy^3 + 2z^2)\mathbf{i} + (9x^2y^2)\mathbf{j} + (4xz + 1)\mathbf{k}$$

defined on all of 3-space:

(a) (3 points)

Prove that the vector field is conservative by demonstrating that its curl is identically 0.

(b) (4 points)

Figure out the generating scalar function f such that $\bigtriangledown f = \mathbf{F}$.

(c) (3 points)

Calculate the line integral from the point $(0, 0, 0)$ to the point $(1, 1, 1)$ along any path using any method you wish.

9. (10 points)

   Evaluate the integral $\int_{-\infty}^{\infty} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$.

   (Note - You must provide a formal evaluation of any integral. For full credit you can't just quote a result from the textbook or from lecture, you must rederive the result. Also, your final answer may be in terms of μ and σ.)

10. (10 points)

Prove the identity:

$$\int_0^x \int_0^v \int_0^u f(t)dtdudv = \frac{1}{2}\int_0^x (x-t)^2 f(t)dt$$

Hint - Switch the order of integration.