Math 2280 - Exam 3

University of Utah

Spring 2014

Name: ____

This is a 50 minute exam. Please show all your work, as a worked problem is required for full points, and partial credit may be rewarded for some work in the right direction.

Things You Might Want to Know

Definitions

$$\mathcal{L}(f(t)) = \int_0^\infty e^{-st} f(t) dt.$$

$$f(t) * g(t) = \int_0^t f(\tau) g(t-\tau) d\tau.$$

Laplace Transforms

$$\mathcal{L}(t^n) = \frac{n!}{s^{n+1}}$$
$$\mathcal{L}(e^{at}) = \frac{1}{s-a}$$
$$\mathcal{L}(\sin(kt)) = \frac{k}{s^2 + k^2}$$
$$\mathcal{L}(\cos(kt)) = \frac{s}{s^2 + k^2}$$
$$\mathcal{L}(\delta(t-a)) = e^{-as}$$
$$\mathcal{L}(u(t-a)f(t-a)) = e^{-as}F(s).$$

Translation Formula

$$\mathcal{L}(e^{at}f(t)) = F(s-a).$$

Derivative Formula

$$\mathcal{L}(x^{(n)}) = s^n X(s) - s^{n-1} x(0) - s^{n-2} x'(0) - \dots - s x^{(n-2)}(0) - x^{(n-1)}(0).$$

1. (10 Points) Calculating a Laplace Transform

Calculate the Laplace transform of the function

$$f(t) = t^2$$

using the definition of the Laplace transform, and state the domain of the transform.

2. (10 Points) Inverse Laplace Transforms

Calculate the inverse Laplace transform of the rational function

$$F(s) = \frac{s+5}{s^2 - 2s - 3}.$$

3. (15 Points) Convolutions

Calculate the the convolution f(t) * g(t) of the following functions:

$$f(t) = t, \quad g(t) = t^2.$$

4. (15 Points) Singular Points

Determine if the point x = 0 is an ordinary point, a regular singular point, or an irregular singular point for the differential equation

$$x^{2}y'' - xy' - 3\cos(x)y = 0.$$

If x = 0 is a regular singular point find the roots of the indicial equation, and state whether we're guaranteed two, or just one, linearly independent Frobenius series solution.

5. (20 Points) *Differential Equations and Laplace Transforms* Find the solution to the initial value problem

$$x'' - 4x' + 4x = \delta(t) + \delta(t - 3),$$
$$x(0) = x'(0) = 0.$$

More room for Problem 5, if you need it.

6. (30 Points) Power Series Solutions

Find the general solution to the ordinary differential equation

$$y'' + xy' + y = 0$$

using power series methods.

Note - The product of all the odd numbers up to and including (2n + 1) is written (2n + 1)!!. The product of all the even numbers up to and including 2n is written $2^n(n!)$.

More room for Problem 6, if you need it.