# Math 2280 - Exam 2

University of Utah

Spring 2014

**Name:** _____

This is a 50 minute exam. Please show all your work, as a worked problem is required for full points, and partial credit may be rewarded for some work in the right direction.

1. (5 points) *System Conversion*

   Convert the following linear differential equation

$$x^{(4)} - 3x^{(3)} + 12x'' - 15x' + 8x = e^t$$

   into an equivalent system of first-order linear differential equations.

2. (10 points) *Particular Solutions*

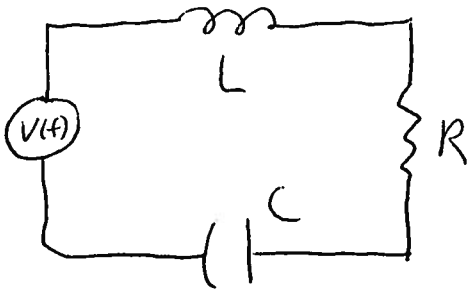
   Using the method of undetermined coefficients, provide the form of the particular solution to the linear differential equation

$$x^{(3)} - 6x'' + 11x' - 6x = t^2 e^t \sin(2t).$$

   You do not need to solve for the undetermined coefficients.

3. (40 points) *LRC Circuits*

For the LRC circuit:




with $V(t) = 13\cos(2t)$, $L = 1$, $R = 2$, and $C = 1$

(a) (5 points) What is the second-order linear differential equation that models the behavior of this system?

(b) (10 points) Find a a particular solution to this differential equation.

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(c) (10 points) What is the general solution to this differential equation?

(d) (5 points) Is the system overdamped, underdamped, or critically damped?

(e) (10 points) What is the current $I(t)$ in the circuit if $I(0) = 5$ and $I'(0) = 8$.

4. (25 points) *Linear Systems of Differential Equations*

   Find the general solution to the system of differential equations:

$$\begin{array}{rcrcrcr} x_1' & = & 3x_1 & + & x_2 & + & x_3 \\ x_2' & = & -5x_1 & - & 3x_2 & - & x_3 \\ x_3' & = & 5x_1 & + & 5x_2 & + & 3x_3 \end{array}$$

   *Hint* - One of the eigenvalues is $\lambda = 3$.

More room for problem whatever, if you need it.

5. (20 points) *Multiple Eigenvalue Systems*[1]

   Find the general solution to the system of differential equations

$$\mathbf{x}' = \begin{pmatrix} 1 & -3 \\ 3 & 7 \end{pmatrix} \mathbf{x}$$

[1]The title of this problem is a hint.