Math 2280 - Exam 1

University of Utah

Spring 2014

Name: ____

This is a 50 minute exam. Please show all your work, as a worked problem is required for full points, and partial credit may be rewarded for some work in the right direction.

- 1. (15 Points) Differential Equation Basics
 - (a) (5 points) What is the order of the differential equation given be-low?¹

$$x^2y^3y^{(4)} - e^{x^2}y^2y' + 3x^{y''} = 42$$

(b) (5 points) Is the differential equation given below linear?

$$y^{(3)} + y^3 = x + 3$$

(c) (5 points) On what intervals are we guaranteed a unique solution exists for the differential equation below?

$$(x+1)y' + e^{2x}y = \frac{x^2+2}{x-1}$$

¹Extra credit - Solve this differential equation! Just kidding. Do not attempt to solve it.

2. (10 points) Phase Diagrams

Find the critical points for the autonomous equation:

$$\frac{dP}{dt} = kP(P-M)(H-P),$$

where k, M, H > 0 and M > H. Draw the corresponding phase diagram, and indicate if the critical points are stable, unstable, or semistable.

3. (20 Points) *Separable Equations*Solve the initial value problem

$$\frac{dP}{dt} = 5P^2,$$

where P(0) = 2.

Why would this be an example of a "doomsday" equation? According to this differential equation, when is "doomsday"?

4. (35 points) Linear Differential Equations

(a) (20 points) Solve the initial value problem

$$y' + 4xy - 3y + 3e^{-2x^2} = 0,$$

 $y(0) = 4.$

(b) (15 points) Find the general solution to the differential equation

$$y'' - y' - 6y = 0.$$

5. (20 points) Euler's Method

Use Euler's method with step size h=1 to estimate y(2) , where y(x) is the solution to the initial value problem

$$\frac{dy}{dx} = 2x + 3y,$$
$$y(0) = 2.$$