Math 2280 - Assignment 8

Dylan Zwick

Spring 2014

Section 5.5 - 1, 7, 9, 18, 24

Section 5.6 - 1, 6, 10, 17, 19

Matrix Exponentials and Linear Systems

5.5.1 - Find a fundamental matrix for the system below, and then find a solution satisfying the given initial conditions.

$$\mathbf{x}' = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 3 \\ -2 \end{pmatrix}.$$

More room for Problem 5.5.1, if you need it.

5.5.7 - Find a fundamental matrix for the system below, and then find a solution satisfying the given initial conditions.

$$\mathbf{x}' = \begin{pmatrix} 5 & 0 & -6 \\ 2 & -1 & -2 \\ 4 & -2 & -4 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}.$$

More room for Problem 5.5.7, if you need it.

5.5.9 - Compute the matrix exponential $e^{\mathbf{A}t}$ for the system $\mathbf{x}' = \mathbf{A}\mathbf{x}$ below.

$$\begin{array}{rcrcr} x_1' & = & 5x_1 & - & 4x_2 \\ x_2' & = & 2x_1 & - & x_2 \end{array}$$

More room for Problem 5.5.9, if you need it.

5.5.18 - Compute the matrix exponential $e^{\mathbf{A}t}$ for the system $\mathbf{x}' = \mathbf{A}\mathbf{x}$ below.

$$\begin{aligned} x_1' &= 4x_1 + 2x_2 \\ x_2' &= 2x_1 + 4x_2 \end{aligned}$$

More room for Problem 5.5.18, if you need it.

5.5.24 - Show that the matrix $\mathbf{A}$ is nilpotent and then use this fact to find the matrix exponential $e^{\mathbf{A}t}$.

$$\mathbf{A} = \begin{pmatrix} 3 & 0 & -3 \\ 5 & 0 & 7 \\ 3 & 0 & -3 \end{pmatrix}$$

More room for Problem 5.5.24, if you need it.

Nonhomogeneous Linear Systems

5.6.1 - Apply the method of undetermined coefficients to find a particular solution to the system below.

$$\begin{aligned} x' &= x + 2y + 3 \\ y' &= 2x + y - 2 \end{aligned}$$

More room for Problem 5.6.1, if you need it.

5.6.6 - Apply the method of undetermined coefficients to find a particular solution to the system below.

$$\begin{array}{rcrcrcl} x' & = & 9x & + & y & + & 2e^t \\ y' & = & -8x & - & 2y & + & te^t \end{array}$$

More room for Problem 5.6.6, if you need it.

5.6.10 - Apply the method of undetermined coefficients to find a particular solution to the system below.

$$\begin{array}{rcrcrcl} x' &=& x &-& 2y & & \\ y' &=& 2x &-& y &+& e^t \sin t \end{array}$$

More room for Problem 5.6.10, if you need it.

5.6.17 - Use the method of variation of parameters to solve the initial value problem

$$\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{f}(t),$$

$$\mathbf{x}(a) = \mathbf{x}_a.$$

The matrix exponential $e^{\mathbf{A}t}$ is given.

$$\mathbf{A} = \begin{pmatrix} 6 & -7 \\ 1 & -2 \end{pmatrix}, \quad \mathbf{f}(t) = \begin{pmatrix} 60 \\ 90 \end{pmatrix}, \quad \mathbf{x}(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix},$$

$$e^{\mathbf{A}t} = \frac{1}{6} \begin{pmatrix} -e^{-t} + 7e^{5t} & 7e^{-t} - 7e^{5t} \\ -e^{-t} + e^{5t} & 7e^{-t} - e^{5t} \end{pmatrix}$$

More room for Problem 5.6.17, if you need it.

5.6.19 - Use the method of variation of parameters to solve the initial value problem

$$\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{f}(t),$$

$$\mathbf{x}(a) = \mathbf{x}_a.$$

The matrix exponential $e^{\mathbf{A}t}$ is given.

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 2 & -2 \end{pmatrix}, \quad \mathbf{f}(t) = \begin{pmatrix} 180t \\ 90 \end{pmatrix}, \quad \mathbf{x}(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix},$$

$$e^{\mathbf{A}t} = \frac{1}{5} \begin{pmatrix} e^{-3t} + 4e^{2t} & -2e^{-3t} + 2e^{2t} \\ -2e^{-3t} + 2e^{2t} & 4e^{-3t} + e^{2t} \end{pmatrix}.$$

More room for Problem 5.6.19, if you need it.