Math 2280 - Assignment 7

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Section 5.1 - 1, 7, 15, 21, 27

Section 5.2 - 1, 9, 15, 21, 39

Section 5.4 - 1, 8, 15, 25, 33

Section 5.1 - Matrices and Linear Systems

5.1.1 - Let

$$\mathbf{A} = \begin{pmatrix} 2 & -3 \\ 4 & 7 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 3 & -4 \\ 5 & 1 \end{pmatrix}.$$

Find

(a) $2\mathbf{A} + 3\mathbf{B}$;

(b) $3\mathbf{A} - 2\mathbf{B}$;

(c) $\mathbf{AB}$;

(d) $\mathbf{BA}$.

More room for Problem 5.1.1, if you need it.

5.1.7 - For the matrices

$$\mathbf{A} = \begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix},$$

Calculate **AB**, and then compute the determinants of the matrices **A** and **B** above. Are your results consistent with the theorem to the effect that

$$det(\mathbf{AB}) = det(\mathbf{A})det(\mathbf{B})$$

for any two square matrices **A** and **B** of the same order?

5.1.15 - Write the system below in the form $\mathbf{x}' = \mathbf{P}(t)\mathbf{x} + \mathbf{f}(t)$.

$$\begin{array}{rcrcrcr} x' & = & & & y & + & z \\ y' & = & x & & & + & z \\ z' & = & x & + & y & & \end{array}$$

5.1.21 For the system below, first verify that the given vectors are solutions of the system. Then use the Wronskian to show that they are linearly independent. Finally, write the general solution of the system.

$$\mathbf{x}' = \begin{pmatrix} 4 & 2 \\ -3 & -1 \end{pmatrix} \mathbf{x};$$

$$\mathbf{x}_1 = \begin{pmatrix} 2e^t \\ -3e^t \end{pmatrix} \quad \mathbf{x}_2 = \begin{pmatrix} e^{2t} \\ -e^{2t} \end{pmatrix}.$$

5.1.27 For the system below, first verify that the given vectors are solutions of the system. Then use the Wronskian to show that they are linearly independent. Finally, write the general solution of the system.

$$\mathbf{x}' = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \mathbf{x};$$

$$\mathbf{x}_1 = e^{2t} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{x}_2 = e^{-t} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \quad \mathbf{x}_3 = e^{-t} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}.$$

More room for Problem 5.1.27, if you need it.

The Eigenvalue Method for Homogeneous Systems

5.2.1 - Apply the eigenvalue method to find the general solution to the system below. Use a computer or graphing calculator to construct a direction field and typical solution curves for the system.

$$\begin{aligned} x_1' &= x_1 + 2x_2 \\ x_2' &= 2x_1 + x_2 \end{aligned}$$

More room for Problem 5.2.1, if you need it.

5.2.9 - Apply the eigenvalue method to find the particular solution to the initial value problem below. Use a computer or graphing calculator to construct a direction field and typical solution curves for the system.

$$\begin{array}{rcrcr} x_1' & = & 2x_1 & - & 5x_2 \\ x_2' & = & 4x_1 & - & 2x_2 \end{array}$$

$$x_1(0) = 2, \quad x_2(0) = 3.$$

More room for Problem 5.2.9, if you need it.

5.2.15 - Apply the eigenvalue method to find the general solution to the system below. Use a computer or graphing calculator to construct a direction field and typical solution curves for the system.

$$\begin{aligned} x_1' &= 7x_1 - 5x_2 \\ x_2' &= 4x_1 + 3x_2 \end{aligned}$$

More room for Problem 5.2.15, if you need it.

5.2.21 - The eigenvalues of the system below can be found by inspection and factoring. Apply the eigenvalue method to find a general solution to the system.

$$\begin{array}{rcrcrcr} x_1' & = & 5x_1 & & & - & 6x_3 \\ x_2' & = & 2x_1 & - & x_2 & - & 2x_3 \\ x_3' & = & 4x_1 & - & 2x_2 & - & 4x_3 \end{array}$$

More room for Problem 5.2.21 if you need it.

5.2.39 For the matrix given below the zeros of the matrix make its characteristic polynomial easy to calculate. Find the general solution of $\mathbf{x}' = A\mathbf{x}$.

$$A = \begin{pmatrix} -2 & 0 & 0 & 9 \\ 4 & 2 & 0 & -10 \\ 0 & 0 & -1 & 8 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

More room for Problem 5.2.39 if you need it.

Section 5.4 - Multiple Eigenvalue Solutions

5.4.1 - Find a general solution to the system of differential equations below.

$$\mathbf{x}' = \begin{pmatrix} -2 & 1 \\ -1 & -4 \end{pmatrix} \mathbf{x}$$

5.4.8 Find a general solution to the system of differential equations below.

$$\mathbf{x}' = \begin{pmatrix} 25 & 12 & 0 \\ -18 & -5 & 0 \\ 6 & 6 & 13 \end{pmatrix} \mathbf{x}$$

More room for Problem 5.4.8, if you need it.

5.4.15 - Find a general solution to the system of differential equations below.

$$\mathbf{x}' = \begin{pmatrix} -2 & -9 & 0 \\ 1 & 4 & 0 \\ 1 & 3 & 1 \end{pmatrix} \mathbf{x}$$

More room for Problem 5.4.15, if you need it.

5.4.25 - Find a general solution to the system of differential equations below. The eigenvalues of the matrix are given.

$$\mathbf{x}' = \begin{pmatrix} -2 & 17 & 4 \\ -1 & 6 & 1 \\ 0 & 1 & 2 \end{pmatrix} \mathbf{x}; \quad \lambda = 2, 2, 2.$$

More room for Problem 5.4.25, if you need it.

5.4.33 - The characteristic equation of the coefficient matrix A of the system

$$\mathbf{x}' = \begin{pmatrix} 3 & -4 & 1 & 0 \\ 4 & 3 & 0 & 1 \\ 0 & 0 & 3 & -4 \\ 0 & 0 & 4 & 3 \end{pmatrix} \mathbf{x}$$

is

$$\phi(\lambda) = (\lambda^2 - 6\lambda + 25)^2 = 0.$$

Therefore, A has the repeated complex conjugate pair $3 \pm 4i$ of eigenvalues. First show that the complex vectors

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ i \\ 0 \\ 0 \end{pmatrix} \quad \mathbf{v}_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ i \end{pmatrix}$$[1]

form a length 2 chain $\{\mathbf{v}_1, \mathbf{v}_2\}$ associated with the eigenvalue $\lambda = 3 - 4i$. Then calculate the real and imaginary parts of the complex-valued solutions

$$\mathbf{v}_1 e^{\lambda t} \quad \text{and} \quad (\mathbf{v}_1 t + \mathbf{v}_2) e^{\lambda t}$$

to find four independent real-valued solutions of $\mathbf{x}' = A\mathbf{x}$.

[1]Note in the textbook there's a typo in this vector.

More room for Problem 5.4.33. You’ll probably need it.

Even MORE room for Problem 5.4.33, just in case.