Math 2280 - Assignment 6

Dylan Zwick

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Section 3.8 - 1, 3, 5, 8, 13

Section 4.1 - 1, 2, 13, 15, 22

Section 4.2 - 1, 10, 19, 28

Section 3.8 - Endpoint Problems and Eigenvalues

3.8.1 For the eigenvalue problem

$$y'' + \lambda y = 0; \quad y'(0) = 0, y(1) = 0,$$

first determine whether $\lambda = 0$ is an eigenvalue; then find the positive eigenvalues and associated eigenfunctions.

Solution - First, if $\lambda = 0$ then the solution to the differential equation

$$y'' = 0$$

is

$$y = Ax + B.$$

From this we get $y' = A$, and so if $y'(0) = 0$ we must have $A = 0$. This would mean $y = B$, and if $y(1) = 0$ then $B = 0$. So, only the trivial solution $A = B = 0$ works, and therefore $\lambda = 0$ is *not* an eigenvalue.

For $\lambda > 0$ the characteristic polynomial for our linear differential equation is:

$$r^2 + \lambda = 0,$$

which has roots $r = \pm\sqrt{-\lambda}$. The corresponding solution to our ODE will be:

$$y = A\cos(\sqrt{\lambda}x) + B\sin(\sqrt{\lambda}x).$$

with derivative

$$y' = -A\sqrt{\lambda}\sin(\sqrt{\lambda}x) + B\sqrt{\lambda}\cos(\sqrt{\lambda}x).$$

So, $y'(0) = B\sqrt{\lambda}$, and therefore if $y'(0) = 0$ then we must have $B = 0$, as $\lambda > 0$. So, our solution must be of the form:

$$y = A\cos(\sqrt{\lambda}x).$$

If we plug in $y(1) = 0$ we get:

$$y(1) = A\cos(\sqrt{\lambda}) = 0.$$

If $A \neq 0$ we must have $\cos(\sqrt{\lambda}) = 0$, which is true only if $\sqrt{\lambda} = \frac{\pi}{2} + n\pi$. So, the eigenvalues are:

$$\lambda_n = \left(\pi\left(\frac{1}{2} + n\right)\right)^2, \text{ with } n \in \mathbb{N},$$

and corresponding eigenfunctions

$$y_n = \cos\left(\left(\frac{\pi}{2} + n\pi\right)x\right).$$

3.8.3 Same instructions as Problem 3.8.1, but for the eigenvalue problem:

$$y'' + \lambda y = 0; \quad y(-\pi) = 0, y(\pi) = 0.$$

Solution - If $\lambda = 0$ then, as in Problem 3.8.1, our solution will be of the form:

$$y = Ax + B.$$

This means $y(\pi) = A\pi + B = 0$, and $y(-\pi) = -A\pi + B = 0$. Adding these two equations we get $2B = 0$, which means $B = 0$. If $B = 0$ then $A\pi = 0$, which means $A = 0$. So, the only solution is the trivial solution $A = B = 0$, and therefore $\lambda = 0$ is *not* an eigenvalue.

Now if $\lambda > 0$ then again just as in Problem 3.8.1 we'll have a solution of the form:

$$y(x) = A\cos{(\sqrt{\lambda}x)} + B\sin{(\sqrt{\lambda}x)}.$$

If we plug in our endpoint values we get:

$$y(\pi) = A\cos{(\sqrt{\lambda}\pi)} + B\sin{(\sqrt{\lambda}\pi)} = 0,$$

$$y(-\pi) = A\cos{(-\sqrt{\lambda}\pi)} + B\sin{(-\sqrt{\lambda}\pi)} =$$
$$A\cos{(\sqrt{\lambda}\pi)} - B\sin{(\sqrt{\lambda}\pi)} = 0,$$

where in the second line above we use that $\cos$ is an even function, while $\sin$ is odd.

If we add these two equations together we get:

$$2A\cos{(\sqrt{\lambda}\pi)} = 0.$$

This is true if either $A = 0$ or $\sqrt{\lambda} = \left(\frac{1}{2} + n\right)$. If $\sqrt{\lambda} = \left(\frac{1}{2} + n\right)$ then

$$y(\pi) = B \sin\left(\left(\frac{1}{2} + n\right)\pi\right) = 0.$$

As $\sin\left(\left(\frac{1}{2} + n\right)\pi\right) = \pm 1$ we must have $B = 0$.

On the other hand, if $A = 0$ above then we have:

$$y(\pi) = B \sin(\sqrt{\lambda}\pi).$$

If $B \neq 0$ then we must have $\sqrt{\lambda} = n$. Combining our two results we get that the possible eigenvalues are:

$$\lambda_n = \frac{n^2}{4},$$

for $n \in \mathbb{N}$, and $n > 0$, with corresponding eigenfunctions:

$$y_n(x) = \begin{cases} \cos\left(\frac{n}{2}x\right) & n \textit{ odd} \\ \sin\left(\frac{n}{2}x\right) & n \textit{ even} \end{cases}$$

3.8.5 Same instructions as Problem 3.8.1, but for the eigenvalue problem:

$$y'' + \lambda y = 0; \quad y(-2) = 0, y'(2) = 0.$$

Solution - If $\lambda = 0$ then, just as in Problem 3.8.1, the solution to the ODE will be:

$$y(x) = Ax + B,$$

$$y'(x) = A.$$

If we plug in our endpoint conditions we get $y(-2) = -2A + B = 0$ and $y'(2) = A = 0$. These equations are satisfied if and only if $A = B = 0$, which is the trivial solution. So, $\lambda = 0$ is *not* an eigenvalue.

If $\lambda > 0$ then, just as in Problem 3.8.1, the solution to the ODE will be of the form:

$$y(x) = A\cos(\sqrt{\lambda}x) + B\sin(\sqrt{\lambda}x),$$

with

$$y'(x) = -A\sqrt{\lambda}\sin(\sqrt{\lambda}x) + B\sqrt{\lambda}\cos(\sqrt{\lambda}x).$$

Plugging in the endpoint conditions, and using that $\cos$ is even and $\sin$ is odd, we get:

$$y(-2) = A\cos(-2\sqrt{\lambda}) + B\sin(-2\sqrt{\lambda}) =$$
$$A\cos(2\sqrt{\lambda}) - B\sin(2\sqrt{\lambda}) = 0,$$

$$y'(2) = -A\sqrt{\lambda}\sin(2\sqrt{\lambda}) + B\sqrt{\lambda}\cos(2\sqrt{\lambda}) = 0.$$

If we divide both sides of the second equality by $\sqrt{\lambda}$ we get

$$-A\sin(2\sqrt{\lambda}) + B\cos(2\sqrt{\lambda}) = 0.$$

From these equations we get:

$$A\cos(2\sqrt{\lambda}) = B\sin(2\sqrt{\lambda}) \Rightarrow \frac{A}{B} = \tan(2\sqrt{\lambda}),$$
$$B\cos(2\sqrt{\lambda}) = A\sin(2\sqrt{\lambda}) \Rightarrow \frac{B}{A} = \tan(2\sqrt{\lambda}).$$

So,

$$\frac{A}{B} = \frac{B}{A} \Rightarrow A^2 = B^2.$$

So, either $A = B$ or $A = -B$.

If $A = B$ then $\tan(2\sqrt{\lambda}) = 1$, which means $2\sqrt{\lambda} = \frac{\pi}{4} + n\pi$, and therefore

$$\lambda = \left(\left(\frac{1+4n}{8}\right)\pi\right)^2.$$

If $A = -B$ then $\tan(2\sqrt{\lambda}) = -1$, which means $2\sqrt{\lambda} = \frac{3\pi}{4} + n\pi$, and therefore

$$\lambda = \left(\left(\frac{3+4n}{8}\right)\pi\right)^2.$$

So, the eigenvalues are:

$$\lambda_n = \left(\left(\frac{1+2n}{8} \right) \pi \right)^2$$

with $n \in \mathbb{N}$ and $n > 0$, with corresponding eigenfunctions:

$$y_n = \begin{cases} \cos\left(\left(\frac{1+2n}{8}\right)\pi x\right) + \sin\left(\left(\frac{1+2n}{8}\right)\pi x\right) & n\ even \\ \cos\left(\left(\frac{1+2n}{8}\right)\pi x\right) - \sin\left(\left(\frac{1+2n}{8}\right)\pi x\right) & n\ odd \end{cases}$$

3.8.8 - Consider the eigenvalue problem

$$y'' + \lambda y = 0; \quad y(0) = 0 \quad y(1) = y'(1) \text{ (not a typo).};$$

all its eigenvalues are nonnegative.

(a) Show that $\lambda = 0$ is an eigenvalue with associated eigenfunction $y_0(x) = x$.

(b) Show that the remaining eigenfunctions are given by $y_n(x) = \sin \beta_n x$, where β_n is the nth positive root of the equation $\tan z = z$. Draw a sketch showing these roots. Deduce from this sketch that $\beta_n \approx (2n+1)\pi/2$ when n is large.

Solution -

(a) - If $\lambda = 0$ then the solution to the ODE will be of the form:

$$y(x) = Ax + B,$$

with

$$y'(x) = A.$$

So, $y(0) = B = 0$, and $y(1) = A = y'(1)$. So, any function of the form $y(x) = Ax$ will work, and our eigenfunction for $\lambda = 0$ is:

$$y_0 = x.$$

(b) - For $\lambda > 0$ the solutions will all be of the form:

$$y(x) = A\cos(\lambda x) + B\sin(\lambda x).$$

If we plug in $y(0) = A = 0$ we get the solutions are of the form:

$$y(x) = B \sin(\sqrt{\lambda} x),$$

with

$$y'(x) = B\sqrt{\lambda}\cos(\sqrt{\lambda} x).$$

If we plug in the other endpoint values we get:

$$y(1) = B \sin(\sqrt{\lambda}) = B\sqrt{\lambda}\cos(\sqrt{\lambda}) = y'(1).$$

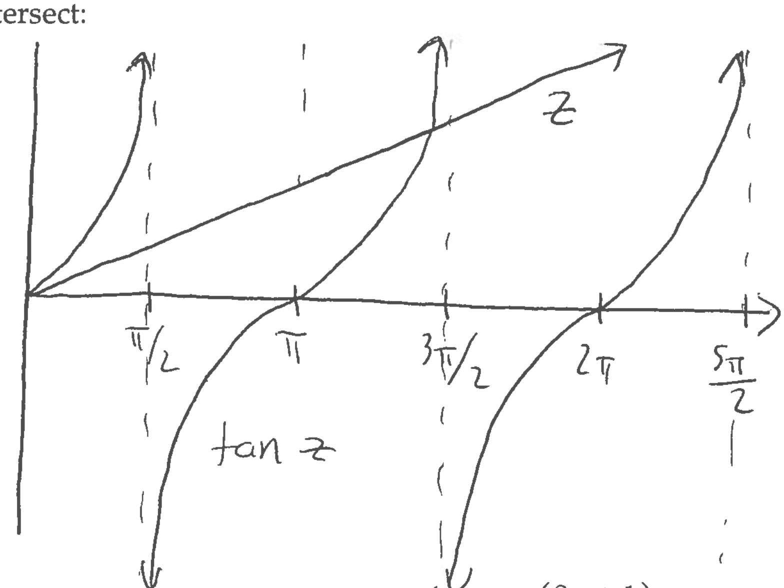
If $B \neq 0$ then we must have:

$$\tan(\sqrt{\lambda}) = \sqrt{\lambda}.$$

So, $\sqrt{\lambda}$ works if it's a root of the equation $\tan z = z$, and if β_n is the nth such root, then the associated eigenfunction is:

$$y_n = \sin(\beta_n x).$$

A sketch of z and $\tan z$ are below. The roots are where they intersect:



As n gets large it occurs at approximately $\left(\frac{2n+1}{2}\right)\pi$.

3.8.13 - Consider the eigenvalue problem

$$y'' + 2y' + \lambda y = 0; \quad y(0) = y(1) = 0.$$

(a) Show that $\lambda = 1$ is not an eigenvalue.

(b) Show that there is no eigenvalue λ such that $\lambda < 1$.

(c) Show that the nth positive eigenvalue is $\lambda_n = n^2\pi^2 + 1$, with associated eigenfunction $y_n(x) = e^{-x}\sin(n\pi x)$.

Solution -

(a) - If $\lambda = 1$ then the characteristic polynomial is:

$$r^2 + 2r + 1 = (r+1)^2,$$

which has roots $r = -1, -1$. So, -1 is a root with multiplicity 2. The corresponding solution to the ODE will be:

$$y(x) = Ae^{-x} + Bxe^{-x}.$$

If we plug in the endpoint values we get:

$$y(0) = A = 0,$$

$$y(1) = Ae^{-1} + Be^{-1} = Be^{-1} = 0.$$

From these we see the only solution is the trivial solution $A = B = 0$, so $\lambda = 1$ is *not* an eigenvalue.

(b) - If $\lambda < 1$ then the characteristic polynomial will be:

$$r^2 + 2r + \lambda,$$

which has roots

$$r = \frac{-2 \pm \sqrt{2^2 - 4(1)\lambda}}{2} = -1 \pm \sqrt{1 - \lambda}.$$

If $\lambda < 1$ then $\sqrt{1-\lambda}$ will be real, and the solution to our ODE will be of the form:

$$y(x) = Ae^{(-1+\sqrt{1-\lambda})x} + Be^{(-1-\sqrt{1-\lambda})x}.$$

Plugging in our endpoint values we get:

$$y(0) = A + B = 0,$$

$$y(1) = Ae^{-1+\sqrt{1-\lambda}} + Be^{-1-\sqrt{1-\lambda}} = 0.$$

From these we get, after a little algebra:

$$A(1 - e^{-2\sqrt{1-\lambda}}) = 0.$$

If $\lambda < 1$ then $e^{-2\sqrt{1-\lambda}} < 1$, and therefore $1 - e^{2\sqrt{1-\lambda}} > 0$. So, for the above equality to be true we must have $A = 0$, which means $B = 0$, and so the only solution is the trivial solution $A = B = 0$. Therefore, no value $\lambda < 1$ is an eigenvalue.

(c) - If $\lambda > 1$ then again using the roots from the quadratic equation in part (b) we get that our solutions will be of the form:

$$y(x) = Ae^{-x}\cos{(\sqrt{\lambda - 1}x)} + Be^{-x}\sin{(\sqrt{\lambda - 1}x)}.$$

If we plug in the endpoint values we get:

$$y(0) = A = 0,$$

and so

$$y(x) = Be^{-x}\sin{(\sqrt{\lambda - 1}x)}.$$

If we plug in our other endpoint value we get:

$$y(1) = Be^{-1}\sin{(\sqrt{\lambda - 1})} = 0.$$

If $B \neq 0$ then we must have $\sin{(\sqrt{\lambda - 1})} = 0$, which is only possible if

$$\sqrt{\lambda - 1} = n\pi,$$

$$\Rightarrow \lambda_n = n^2\pi^2 + 1.$$

So, the eigenvalues are given above, and the corresponding eigenfunctions are:

$$y_n = e^{-x}\sin{(n\pi x)},$$

for $n \in \mathbb{N}$, $n > 0$.

Section 4.1 - First-Order Systems and Applications

4.1.1 - Transform the given differential equation into an equivalent system of first-order differential equations.

$$x'' + 3x' + 7x = t^2.$$

Solution - If we define $x = x_1$ then define:

$$x_1' = x_2,$$

$$x_2' = t^2 - 3x_2 - 7x_1.$$

So, the system is:

$$\begin{array}{rcrcrcr} x_1' & = & & & x_2 & & \\ x_2' & = & -7x_1 & - & 3x_2 & + & t^2 \end{array}.$$

4.1.2 - Transform the given differential equation into an equivalent system of first-order differential equations.

$$x^{(4)} + 6x'' - 3x' + x = \cos 3t.$$

Solution - Define $x = x_1$. Then the equivalent system is:

$$\begin{array}{rcl} x_1' & = & x_2 \\ x_2' & = & x_3 \\ x_3' & = & x_4 \\ x_4' & = & -6x_3 + 3x_2 - x_1 + \cos(3t) \end{array}.$$

4.1.13 - Find the particular solution to the system of differential equations below. Use a computer system or graphing calculator to construct a direction field and typical solution curves for the given system.

$$x' = -2y, \quad y' = 2x; \quad x(0) = 1, y(0) = 0.$$

Solution - If we differentiate $y' = 2x$, we get $y'' = 2x' = -4y$. So, we have the differential equation:

$$y'' + 4y = 0.$$

The solution to this ODE is:

$$y(t) = A\cos(2t) + B\sin(2t).$$

Now,

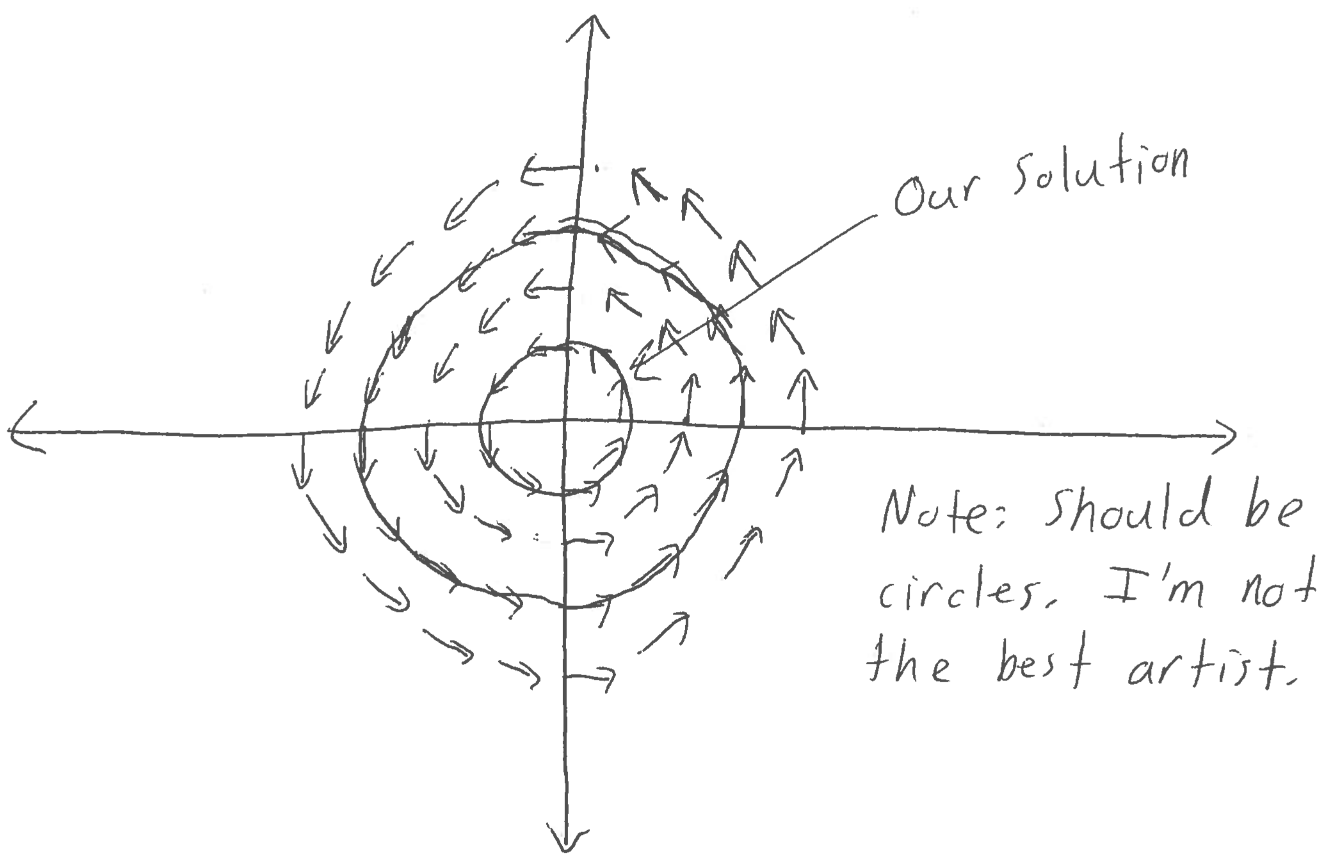
$$x(t) = \frac{1}{2}y' = \frac{1}{2}(-2A\sin(2t) + 2B\cos(2t)) = -A\sin(2t) + B\cos(2t).$$

If we plug in $x(0) = B = 1$ and $y(0) = A = 0$ we get:

$$x(t) = \cos(2t)$$
$$y(t) = \sin(2t).$$

More room, if necessary, for Problem 4.1.13.

Direction Field



4.1.15 - Find the general solution to the system of differential equations below. Use a computer system or graphing calculator to construct a direction field and typical solution curves for the given system.

$$x' = \frac{1}{2}y, \quad y' = -8x.$$

Solution - If we differentiate $y' = -8x$ we get $y'' = -8x' = -4y$. So, our ODE is:

$$y'' + 4y = 0.$$

The solution to this ODE is:

$$y(t) = A\cos(2t) + B\sin(2t).$$

The function $x(t)$ is:

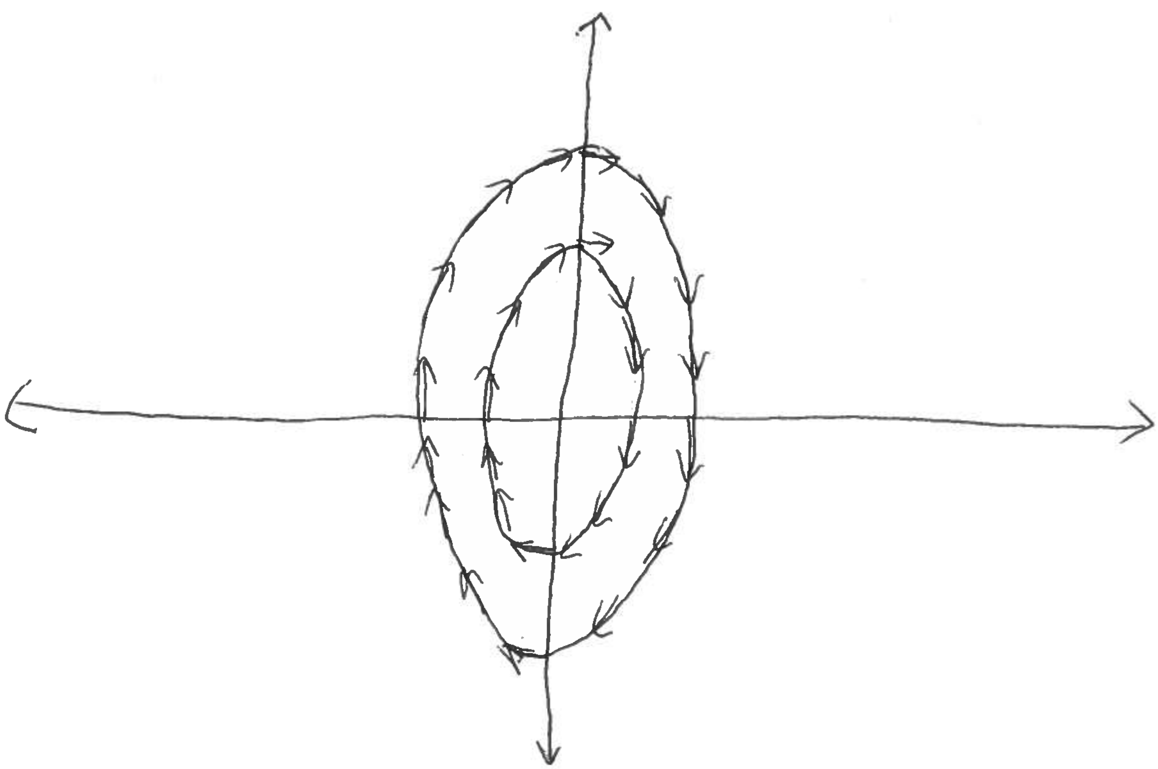
$$x(t) = -\frac{1}{8}y'(t) = -\frac{A}{4}\sin(2t) + \frac{B}{4}\cos(2t).$$

So, the general solution to this system of ODEs is:

$$x(t) = -\frac{A}{4}\sin(2t) + \frac{B}{4}\cos(2t)$$
$$y(t) = A\cos(2t) + B\sin(2t).$$

More room, if necessary, for Problem 4.1.15.

Direction Field



4.1.22 (a) - Beginning with the general solution of the system from Problem 13, calculate $x^2 + y^2$ to show that the trajectories are circles.

(b) - Show similarly that the trajectories of the system from Problem 15 are ellipses of the form $16x^2 + y^2 = C^2$.

(a) - The general solution to the system of ODEs from Problem 4.1.13 is:

$$x(t) = -A\sin(2t) + B\cos(2t)$$
$$y(t) = A\cos(2t) + B\sin(2t).$$

From these we get:

$$x(t)^2 + y(t)^2 = (-A\sin(2t) + B\cos(2t))^2 + (A\cos(2t) + B\sin(2t))^2$$
$$= A^2\sin^2(2t) - 2AB\sin(2t)\cos(2t) + B^2\cos^2(2t) + A^2\cos^2(2t) + 2AB\sin(2t)\cos(2t) + B^2\sin^2(2t)$$
$$= A^2 + B^2.$$

So, circles.

(b) - The general solution to the system of ODEs from Problem 4.1.15 is:

$$x(t) = -\frac{A}{4}\sin(2t) + \frac{B}{4}\cos(2t)$$
$$y(t) = A\cos(2t) + B\sin(2t).$$

So,

$$16x(t)^2 = A^2\sin^2(2t) - 2AB\sin(2t)\cos(2t) + B^2\cos^2(2t),$$
$$y(t)^2 = A^2\cos^2(2t) + 2AB\sin(2t)\cos(2t) + B^2\sin^2(2t).$$

Combining these we get $16x(t)^2 + y(t)^2 = A^2 + B^2 = C^2$. So, ellipses.

Section 4.2 - The Method of Elimination

4.2.1 - Find a general solution to the linear system below. Use a computer system or graphing calculator to construct a direction field and typical solution curves for the system.

$$\begin{aligned} x' &= -x + 3y \\ y' &= 2y \end{aligned}$$

Solution - The differential equation

$$y' = 2y$$

has the solution

$$y(t) = Ae^{2t}.$$

So,

$$x' = -x + 3Ae^{2t} \Rightarrow x' + x = 3Ae^{2t}.$$

This is a first-order linear ODE. Its integrating factor is:

$$\rho(t) = e^{\int 1dt} = e^t.$$

Multiplying both sides by this integrating factor our linear ODE becomes:

$$\frac{d}{dt}\left(e^t x\right) = 3Ae^{3t}.$$

Integrating both sides we get:

$$e^t x = Ae^{3t} + B$$
$$\Rightarrow x = Ae^{2t} + Be^{-t}.$$

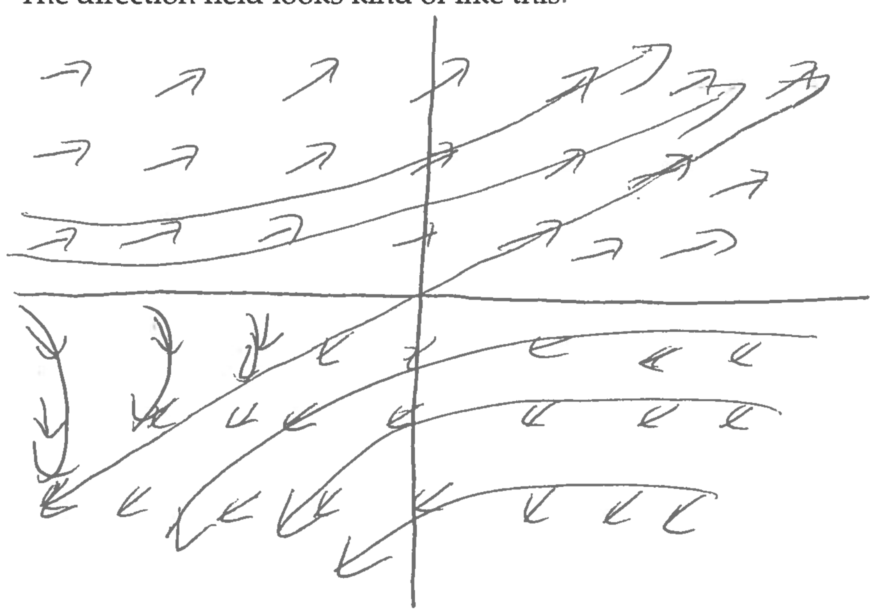
So, the general solution to this system is:

$$x(t) = Ae^{2t} + Be^{-t},$$
$$y(t) = Ae^{2t}.$$

We can write this in vector form as:

$$\begin{pmatrix} x \\ y \end{pmatrix} = A \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t} + B \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-t}.$$

The direction field looks kind of like this:



4.2.10 Find a particular solution to the given system of differential equations that satisfies the given initial conditions.

$$x' + 2y' = 4x + 5y,$$

$$2x' - y' = 3x;$$

$$x(0) = 1, y(0) = -1.$$

Solution - If we add 2 times the second equation to the first we get:

$$5x' = 10x + 5y.$$

If we subtract 2 times the first equation from the second we get:

$$-5y' = -5x - 10y \Rightarrow 5y' = 5x + 10y.$$

Differentiating $5x' = 10x + 5y$ and plugging in $5y' = 5x + 10y$ we get:

$$5x'' = 10x' + 5y' = 10x' + (5x + 10y)$$

$$\Rightarrow 5x'' = 10x' + (5x + 10x' - 20x)$$

$$\Rightarrow 5x'' = 20x' - 15x$$

$$\Rightarrow x'' = 4x' - 3x.$$

The linear homogeneous differential equation $x'' - 4x' + 3x = 0$ has characteristic equation:

$$r^2 - 4r + 3 = (r - 3)(r - 1).$$

So, the roots are $r = 3, 1$, and the general solution to the ODE is:

$$x(t) = c_1 e^{3t} + c_2 e^{t}.$$

From the equation $5y' = 5x + 10y$ we get $y' = x + 2y$, and therefore:

$$y' - 2y = c_1 e^{3t} + c_2 e^{t}.$$

If we multiply both sides by the integrating factor e^{-2t} we get:

$$\frac{d}{dt}\left(e^{-2t} y\right) = c_1 e^{t} + c_2 e^{-t}.$$

Integrating both sides we get:

$$e^{-2t} y = c_1 e^{t} - c_2 e^{-t} + C,$$

and so:

$$y(t) = c_1 e^{3t} - c_2 e^{t} + C e^{2t}.$$

Plugging this into any of the equations in our system gives us $C = 0$. So,

$$y(t) = c_1 e^{3t} - c_2 e^{t}.$$

We can write this solution in matrix form as:

$$\mathbf{x}(t) = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{3t} + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{t}.$$

If we plug in

$$\mathbf{x}(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

we can see immediately that $c_1 = 0$ and $c_2 = 1$. So, the solution to our initial value problem is:

$$\mathbf{x}(t) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^t.$$

4.2.19 Find a general solution to the given system of differential equations.

$$x' = 4x - 2y,$$

$$y' = -4x + 4y - 2z,$$

$$z' = -4y + 4z.$$

Solution - If we differentiate the first equation we get:

$$x'' = 4x' - 2y' = 4x' - 2(-4x + 4y - 2z)$$

$$\Rightarrow x'' = 4x' + 8x - 8y + 4z.$$

Differentiating again we get:

$$x^{(3)} = 4x'' + 8x' - 8y' + 4z' = 4x'' + 8x' - 8y' + 4(-4y + 4z)$$

$$\Rightarrow x^{(3)} = 4x'' + 8x' - 8y' - 16y + 16z$$

$$\Rightarrow x^{(3)} = 4x'' + 8x' - 8y' - 16y + 8(-y' - 4x + 4y)$$

$$\Rightarrow x^{(3)} = 4x'' + 8x' - 16y' + 16y - 32x$$

$$\Rightarrow x^{(3)} = 4x'' + 8x' - 8(4x' - x'') + 8(4x - x') - 32x$$

$$\Rightarrow x^{(3)} = 12x'' - 32x' \Rightarrow x^{(3)} - 12x'' + 32x' = 0.$$

The characteristic equation for this ODE is

$$r^3 - 12r^2 + 32r = r(r-8)(r-4).$$

So,

$$x(t) = c_1 + c_2 e^{8t} + c_3 e^{4t}.$$

From this we get:

$$x'(t) = 8c_2 e^{8t} + 4c_3 e^{4t}$$

and

$$y(t) = 2x - \frac{1}{2}x' = 2c_1 - 2c_2 e^{8t}.$$

Finally,

$$z(t) = -2x'(t) + 2y(t) - \frac{1}{2}y'(t) = 2c_1 + 2c_2 e^{8t} - 2c_3 e^{4t}.$$

So,

$$x(t) = c_1 + c_2 e^{8t} + c_3 e^{4t},$$

$$y(t) = 2c_1 - 2c_2 e^{8t},$$

$$z(t) = 2c_1 + 2c_2 e^{8t} - 2c_3 e^{4t}.$$

4.2.28 For the system below first calculate the operational determinant to determine how many arbitrary constants should appear in a general solution. Then attempt to solve the system explicitly so as to find such a general solution.

$$\begin{array}{rcccl} (D^2+D)x & + & D^2y & = & 2e^{-t} \\ (D^2-1)x & + & (D^2-D)y & = & 0 \end{array}$$

Solution - The operational determinant of the system above is:

$$(D^2+D)(D^2-D) - D^2(D^2-1) = D^4 - D^3 + D^3 - D^2 - D^4 + D^2 = 0.$$

So, there are 0(!) arbitrary constants. How is this possible? Well, if we subtract the second relation from the first we get:

$$(D+1)x + Dy = 2e^{-t}$$

$$\Rightarrow Dy = 2e^{-t} - (D+1)x$$

$$\Rightarrow D^2y = -2e^{-t} - (D^2+D)x$$

$$\Rightarrow (D^2+D)x + D^2y = -2e^{-t}.$$

However, this cannot be, as our first relation above is:

$$(D^2+D)x + D^2y = 2e^{-t},$$

and $2e^{-t} \neq -2e^{-t}$. So, there is *no solution* to the system.