# Math 2280 - Assignment 6

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**Section 3.8** - 1, 3, 5, 8, 13

**Section 4.1** - 1, 2, 13, 15, 22

**Section 4.2** - 1, 10, 19, 28

# Section 3.8 - Endpoint Problems and Eigenvalues

**3.8.1** For the eigenvalue problem

$$y'' + \lambda y = 0; \quad y'(0) = 0, y(1) = 0,$$

first determine whether $\lambda = 0$ is an eigenvalue; then find the positive eigenvalues and associated eigenfunctions.

**3.8.3** Same instructions as Problem 3.8.1, but for the eigenvalue problem:

$$y'' + \lambda y = 0; \quad y(-\pi) = 0, y(\pi) = 0.$$

More room for Problem 3.8.3 if you need it.

**3.8.5** Same instructions as Problem 3.8.1, but for the eigenvalue problem:

$$y'' + \lambda y = 0; \quad y(-2) = 0, y'(2) = 0.$$

More room for Problem 3.8.5 if you need it.

**3.8.8** - Consider the eigenvalue problem

$$y'' + \lambda y = 0; \quad y(0) = 0 \quad y(1) = y'(1) \text{ (not a typo).;}$$

all its eigenvalues are nonnegative.

**(a)** Show that $\lambda = 0$ is an eigenvalue with associated eigenfunction $y_0(x) = x$.

**(b)** Show that the remaining eigenfunctions are given by $y_n(x) = \sin \beta_n x$, where $\beta_n$ is the $n$th positive root of the equation $\tan z = z$. Draw a sketch showing these roots. Deduce from this sketch that $\beta_n \approx (2n+1)\pi/2$ when $n$ is large.

More room, if necessary, for Problem 3.8.8.

**3.8.13** - Consider the eigenvalue problem

$$y'' + 2y' + \lambda y = 0; \quad y(0) = y(1) = 0.$$

**(a)** Show that $\lambda = 1$ is not an eigenvalue.

**(b)** Show that there is no eigenvalue $\lambda$ such that $\lambda < 1$.

**(c)** Show that the $n$th positive eigenvalue is $\lambda_n = n^2\pi^2 + 1$, with associated eigenfunction $y_n(x) = e^{-x}\sin(n\pi x)$.

More room, if necessary, for Problem 3.8.13.

# Section 4.1 - First-Order Systems and Applications

**4.1.1** - Transform the given differential equation into an equivalent system of first-order differential equations.

$$x'' + 3x' + 7x = t^2.$$

**4.1.2** - Transform the given differential equation into an equivalent system of first-order differential equations.

$$x^{(4)} + 6x'' - 3x' + x = \cos 3t.$$

**4.1.13** - Find the particular solution to the system of differential equations below. Use a computer system or graphing calculator to construct a direction field and typical solution curves for the given system.

$$x' = -2y, \quad y' = 2x; \quad x(0) = 1, y(0) = 0.$$

More room, if necessary, for Problem 4.1.13.

**4.1.15** - Find the general solution to the system of differential equations below. Use a computer system or graphing calculator to construct a direction field and typical solution curves for the given system.

$$x' = \frac{1}{2}y, \quad y' = -8x.$$

More room, if necessary, for Problem 4.1.15.

**4.1.22 (a)** - Beginning with the general solution of the system from Problem 13, calculate $x^2 + y^2$ to show that the trajectories are circles.

**(b)** - Show similarly that the trajectories of the system from Problem 15 are ellipses of the form $16x^2 + y^2 = C^2$.

More room, if necessary, for Problem 4.1.22.

## Section 4.2 - The Method of Elimination

**4.2.1** - Find a general solution to the linear system below. Use a computer system or graphing calculator to construct a direction field and typical solution curves for the system.

$$\begin{aligned} x' &= -x + 3y \\ y' &= \quad\;\; 2y \end{aligned}$$

More room for Problem 4.2.1, if you need it.

**4.2.10** Find a particular solution to the given system of differential equations that satisfies the given initial conditions.

$$x' + 2y' = 4x + 5y,$$

$$2x' - y' = 3x;$$

$$x(0) = 1, y(0) = -1.$$

More room for Problem 4.2.10, if you need it.

**4.2.19** Find a general solution to the given system of differential equations.

$$x' = 4x - 2y,$$

$$y' = -4x + 4y - 2z,$$

$$z' = -4y + 4z.$$

More room for Problem 4.2.19, if you need it.

**4.2.28** For the system below first calculate the operational determinant to determine how many arbitrary constants should appear in a general solution. Then attempt to solve the system explicitly so as to find such a general solution.

$$\begin{array}{rcrcl} (D^2+D)x & + & D^2y & = & 2e^{-t} \\ (D^2-1)x & + & (D^2-D)y & = & 0 \end{array}$$

More room for Problem 4.2.28, if you need it.