Math 2280 - Assignment 5

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Section 3.4 - 1, 5, 18, 21

Section 3.5 - 1, 11, 23, 28, 35, 47, 56

Section 3.6 - 1, 2, 9, 17, 24

Section 3.7 - 1, 5, 10, 17, 19

Section 3.4 - Mechanical Vibrations

3.4.1 - Determine the period and frequency of the simple harmonic motion of a 4-kg mass on the end of a spring with spring constant $16N/m$.

3.4.5 - Assume that the differential equation of a simple pendulum of length L is $L\theta'' + g\theta = 0$, where $g = GM/R^2$ is the gravitational acceleration at the location of the pendulum (at distance R from the center of the earth; M denotes the mass of the earth).

Two pendulums are of lengths L_1 and L_2 and - when located at the respective distances R_1 and R_2 from the center of the earth - have periods p_1 and p_2. Show that

$$\frac{p_1}{p_2} = \frac{R_1\sqrt{L_1}}{R_2\sqrt{L_2}}.$$

3.4.18 - A mass m is attached to both a spring (with spring constant k) and a dashpot (with dampring constant c). The mass is set in motion with initial position x_0 and initial velocity v_0. Find the position function $x(t)$ and determine whether the motion is overdamped, critically damped, or underdamped. If it is underdamped, write the position function in the form $x(t) = C_1 e^{-pt} \cos(\omega_1 t - \alpha_1)$. Also, find the undamped position function $u(t) = C_0 \cos(\omega_0 t - \alpha_0)$ that would result if the mass on the spring were set in motion with the same initial position and velocity, but with the dashpot disconnected (so $c = 0$). Finally, construct a figure that illustrates the effect of damping by comparing the graphs of $x(t)$ and $u(t)$.

$$m = 2, \quad c = 12, \quad k = 50,$$

$$x_0 = 0, \quad v_0 = -8.$$

More room, if necessary, for Problem 3.4.18.

3.4.21 - Same as problem 3.4.18, except with the following values:

$$m = 1, \quad c = 10, \quad k = 125,$$

$$x_0 = 6, \quad v_0 = 50.$$

More room, if necessary, for Problem 3.4.21.

Section 3.5 - Nonhomogeneous Equations and Undetermined Coefficients

3.5.1 - Find a particular solution, y_p, to the differential equation

$$y'' + 16y = e^{3x}.$$

3.5.11 - Find a particular solution, y_p, to the differential equation

$$y^{(3)} + 4y' = 3x - 1.$$

3.5.23 - Set up the appropriate form of a particular solution y_p, but do not determine the values of the coefficients.[1]

$$y'' + 4y = 3x \cos 2x.$$

[1] Unless you really, really want to.

3.5.28 - Same instructions as Problem 3.5.23, but with the differential equation

$$y^{(4)} + 9y'' = (x^2 + 1)\sin 3x.$$

3.5.35 - Solve the initial value problem

$$y'' - 2y' + 2y = x + 1;$$

$$y(0) = 3, \quad y'(0) = 0.$$

3.5.47 - Use the method of variation of parameters to find a particular solution to the differential equation

$$y'' + 3y' + 2y = 4e^x.$$

3.5.56 - Same instructions as Problem 3.5.47, but with the differential equation

$$y'' - 4y = xe^x.$$

Section 3.6 - Forced Oscillations and Resonance

3.6.1 - Express the solution of the initial value problem

$$x'' + 9x = 10\cos 2t;$$

$$x(0) = x'(0) = 0,$$

as a sum of two oscillations in the form:

$$x(t) = C\cos(\omega_0 t - \alpha) + \frac{F_0/m}{\omega_0^2 - \omega^2}\cos\omega t.$$

More space, if necessary, for Problem 3.6.1.

3.6.2 - Same instructions as Problem 3.6.1, but with the initial value problem:

$$x'' + 4x = 5\sin 3t;$$

$$x(0) = x'(0) = 0.$$

More space, if necessary, for Problem 3.6.2.

3.6.9 - Find the steady periodic solution $x_{sp}(t) = C\cos(\omega t - \alpha)$ of the given equation $mx'' + cx' + kx = F(t)$ with periodic forcing function $F(t)$ of frequency ω. Then graph $x_{sp}(t)$ together with (for comparison) the adjusted forcing function $F_1(t) = F(t)/m\omega$.

$$2x'' + 2x' + x = 3\sin 10t.$$

More space, if necessary, for Problem 3.6.9.

3.6.17 - Suppose we have a forced mass-spring-dashpot system with equation:

$$x'' + 6x' + 45x = 50\cos\omega t.$$

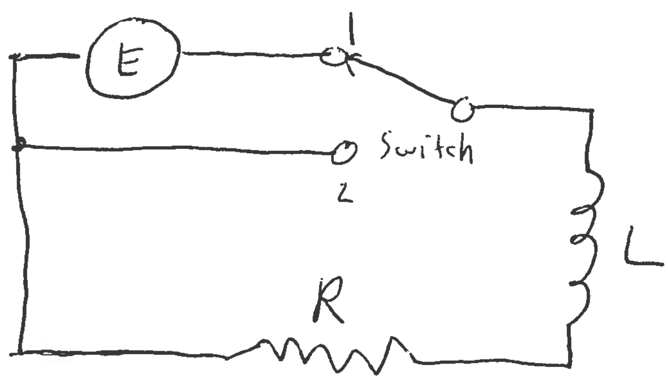
Investigate the possibility of practical resonance of this system. In particular, find the amplitude $C(\omega)$ of steady periodic forced oscillations with frequency ω. Sketch the graph of $C(\omega)$ and find the practical resonance frequency ω (if any).

3.6.24 - A mass on a spring without damping is acted on by the external force $F(t) = F_0 \cos^3 \omega t$. Show that there are *two* values of ω for which resonance occurs, and find both.

Section 3.7 - Electrical Circuits

3.7.1 This problem deals with the RL circuit pictured below. It is a series circuit containing an inductor with an inductance of L henries, a resistor with a resistance of R ohms, and a source of electromotive force (emf), but no capacitor. In this case the equation governing our system is the first-order equation

$$LI' + RI = E(t).$$



Suppose that $L = 5H$, $R = 25\Omega$, and the source E of emf is a battery supplying $100V$ to the circuit. Suppose also that the switch has been in position 1 for a long time, so that a steady current of $4A$ is flowing in the circuit. At time $t = 0$, the switch is thrown to position 2, so that $I(0) = 4$ and $E = 0$ for $t \geq 0$. Find $I(t)$.

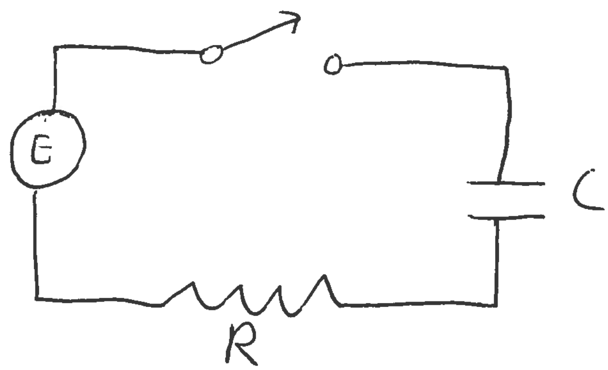
More room, if necessary, for Problem 3.7.1.

3.7.5 - In the circuit from Problem 3.7.1, with the switch in position 1, suppose that $E(t) = 100e^{-10t}\cos 60t$, $R = 20$, $L = 2$, and $I(0) = 0$. Find $I(t)$.

3.7.10 - This problem deals with an RC circuit pictured below, containing a resistor (R ohms), a capacitor (C farads), a switch, a source of emf, but no inductor. This system is governed by the linear first-order differential equation

$$R\frac{dQ}{dt} + \frac{1}{C}Q = E(t).$$

for the charge $Q = Q(t)$ on the capacitor at time t. Note that $I(t) = Q'(t)$.



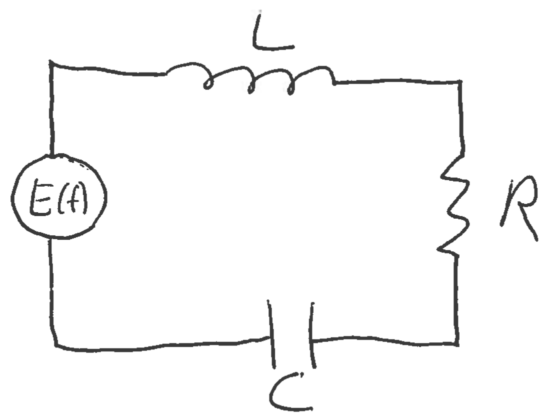
Suppose an emf of voltage $E(t) = E_0 \cos \omega t$ is applied to the RC circuit at time $t = 0$ (with the switch closed), and $Q(0) = 0$. Substitute $Q_{sp}(t) = A \cos \omega t + B \sin \omega t$ in the differential equation to show that the steady periodic charge on the capacitor is

$$Q_{sp}(t) = \frac{E_0 C}{\sqrt{1 + \omega^2 R^2 C^2}} \cos(\omega t - \beta)$$

where $\beta = \tan^{-1}(\omega RC)$.

More room for Problem 3.7.10. You'll probably need it.

3.7.17 For the RLC circuit pictured below find the current $I(t)$ using the given values of R, L, C and $V(t)$, and the given initial values.



$$R = 16\Omega, L = 2H, C = .02F;$$

$$E(t) = 100V; I(0) = 0, Q(0) = 5.$$

More room for Problem 3.7.17 if you need it.

3.7.19 Same instructions as Problem 3.7.17, but with the values:

$$R = 60\Omega,\ L = 2H,\ C = .0025F;$$

$$E(t) = 100e^{-10t}V;\ I(0) = 0, Q(0) = 1.$$