Math 2280 - Assignment 13

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Section 9.1 - 1, 8, 11, 13, 21

Section 9.2 - 1, 9, 15, 17, 20

Section 9.3 - 1, 5, 8, 13, 20

Section 9.1 - Periodic Functions and Trigonometric Series

9.1.1 - Sketch the graph of the function f defined for all t by the given formula, and determine whether it is periodic. If so, find its smallest period.

$$f(t) = \sin 3t.$$

9.1.8 - Sketch the graph of the function f defined for all t by the given formula, and determine whether it is periodic. If so, find its smallest period.

$$f(t) = \sinh \pi t.$$

9.1.11 - The value of a period 2π function $f(t)$ in one full period is given below. Sketch several periods of its graph and find its Fourier series.

$$f(t) = 1, \qquad\qquad -\pi \leq t \leq \pi.$$

9.1.13 - The value of a period 2π function $f(t)$ in one full period is given below. Sketch several periods of its graph and find its Fourier series.

$$f(t) = \begin{cases} 0 & -\pi < t \leq 0 \\ 1 & 0 < t \leq \pi \end{cases}$$

9.1.21 - The value of a period 2π function $f(t)$ in one full period is given below. Sketch several periods of its graph and find its Fourier series.

$$f(t) = t^2, \qquad\qquad -\pi \le t < \pi$$

Section 9.2 - General Fourier Series and Convergence

9.2.1 - The values of a periodic function $f(t)$ in one full period are given below; at each discontinuity the value of $f(t)$ is that given by the average value condition. Sketch the graph of f and find its Fourier series.

$$f(t) = \begin{cases} -2 & -3 < t < 0 \\ 2 & 0 < t < 3 \end{cases}$$

9.2.9 - The values of a periodic function $f(t)$ in one full period are given below; at each discontinuity the value of $f(t)$ is that given by the average value condition. Sketch the graph of f and find its Fourier series.

$$f(t) = t^2, \qquad\qquad -1 < t < 1$$

9.2.15 -

(a) - Suppose that f is a function of period 2π with $f(t) = t^2$ for $0 < t < 2\pi$. Show that

$$f(t) = \frac{4\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{\cos nt}{n^2} - 4\pi \sum_{n=1}^{\infty} \frac{\sin nt}{n}$$

and sketch the graph of f, indicating the value at each discontinuity.

(b) - Deduce the series summations

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

and

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$$

from the Fourier series in part (a).

More room for Problem 9.2.15, if you need it.

9.2.17 -

(a) - Supose that f is a funciton of period 2 with $f(t) = t$ for $0 < t < 2$. Show that

$$f(t) = 1 - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\sin n\pi t}{n}$$

and sketch the graph of f, indicating the value at each discontinuity.

(b) - Substitute an appropriate value of t to deduce *Leibniz's series*

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}.$$

More room for Problem 9.2.17, if you need it.

9.2.20 - Derive the Fourier series given below, and graph the period 2π function to which the series converges.

$$\sum_{n=1}^{\infty} \frac{\cos nt}{n^2} = \frac{3t^2 - 6\pi t + 2\pi^2}{12} \qquad (0 < t < 2\pi)$$

Section 9.3 - Fourier Sine and Cosine Series

9.3.1 - For the given function $f(t)$ defined on the given interval find the Fourier cosine and sine series of f and sketch the graphs of the two extensions of f to which these two series converge.

$$f(t) = 1, \qquad\qquad 0 < t < \pi.$$

More room for Problem 9.3.1, if you need it.

9.3.5 - For the given function $f(t)$ defined on the given interval find the Fourier cosine and sine series of f and sketch the graphs of the two extensions of f to which these two series converge.

$$f(t) = \begin{cases} 0 & 0 < t < 1 \\ 1 & 1 < t < 2 \\ 0 & 2 < t < 3 \end{cases}$$

More room for Problem 9.3.5, if you need it.

9.3.8 - For the given function $f(t)$ defined on the given interval find the Fourier cosine and sine series of f and sketch the graphs of the two extensions of f to which these two series converge.

$$f(t) = t - t^2, \qquad\qquad 0 < t < 1$$

More room for Problem 9.3.8, if you need it.

9.3.13 - Find a formal Fourier series solution to the endpoint value problem

$$x'' + x = t \qquad\qquad x(0) = x(1) = 0.$$

More room for Problem 9.3.13, if you need it.

9.3.20 - Substitute $t = \pi/2$ and $t = \pi$ in the series

$$\frac{1}{24}t^4 = \frac{\pi^2 t^2}{12} - 2\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4} \cos nt + 2\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4}, \quad -\pi < t < \pi,$$

to obtain the summations

$$\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90},$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4} = \frac{7\pi^4}{720},$$

and

$$1 + \frac{1}{3^4} + \frac{1}{5^4} + \frac{1}{7^4} + \cdots = \frac{\pi^4}{96}.$$

More room for Problem 9.3.20, if you need it.