Math 2280 - Practice Exam 4

University of Utah

Spring 2013

Name: _____

This is a 50 minute exam. Please show all your work, as a worked problem is required for full points, and partial credit may be rewarded for some work in the right direction.

Laplace Transforms You May Need

Definition

$$\mathcal{L}(f(t)) = \int_0^\infty e^{-st} f(t) dt.$$

$$\mathcal{L}(e^{at}) = \frac{1}{s-a}$$

$$\mathcal{L}(\sin(kt)) = \frac{k}{s^2+k^2}$$

$$\mathcal{L}(\cos(kt)) = \frac{s}{s^2+k^2}$$

$$\mathcal{L}(\delta(t-a)) = e^{-as}$$

$$\mathcal{L}(u(t-a)f(t-a)) = e^{-as}F(s).$$

1. (15 points) *The Laplace Transform*
   Calculate the Laplace transform of the function

$$f(t) = t^2$$

   using the definition of the Laplace transform.

2. (25 points) *Laplace Transforms and Initial Value Problems*

   Use Laplace transforms to solve the initial value problem

$$x'' - 6x' + 8x = 2$$

$$x(0) = x'(0) = 0.$$

More room for Problem 2, if you need it.

3. (15 points) *Convolutions*

   Calculate the convolution, $f(t) * g(t)$, of the following functions

$$f(t) = t, \qquad g(t) = e^{at}.$$

4. (25 points) *Delta Functions*

   Solve the initial value problem

$$x'' + 2x' + x = \delta(t) - \delta(t-2)$$

$$x(0) = x'(0) = 2.$$

More room for Problem 4, if you need it.

5. (10 points) *The Ratio Test*

   Use the ratio test to determine the radius of convergence of the geometric series

$$\sum_{n=0}^{\infty} x^n$$