Math 2280 - Exam 4

University of Utah

Spring 2013

Name: _____

This is a 50 minute exam. Please show all your work, as a worked problem is required for full points, and partial credit may be rewarded for some work in the right direction.

Things You Might Want to Know

Definitions

$$\mathcal{L}(f(t)) = \int_0^\infty e^{-st} f(t) dt.$$

$$f(t) * g(t) = \int_0^t f(\tau) g(t-\tau) d\tau.$$

Laplace Transforms

$$\mathcal{L}(t^n) = \frac{n!}{s^{n+1}}$$

$$\mathcal{L}(e^{at}) = \frac{1}{s-a}$$

$$\mathcal{L}(\sin(kt)) = \frac{k}{s^2+k^2}$$

$$\mathcal{L}(\cos(kt)) = \frac{s}{s^2+k^2}$$

$$\mathcal{L}(\delta(t-a)) = e^{-as}$$

$$\mathcal{L}(u(t-a)f(t-a)) = e^{-as}F(s).$$

Translation Formula

$$\mathcal{L}(e^{at}f(t)) = F(s-a).$$

Derivative Formula

$$\mathcal{L}(x^{(n)}) = s^n X(s) - s^{n-1}x(0) - s^{n-2}x'(0) - \cdots - sx^{(n-2)}(0) - x^{(n-1)}(0).$$

1. (15 points) *The Laplace Transform*
   Calculate the Laplace transform of the function

$$f(t) = e^{at}$$

   using the definition of the Laplace transform.

2. (25 points) *Laplace Transforms and Initial Value Problems*

   Use Laplace transforms to solve the initial value problem

$$x'' - x' - 2x = 0$$

$$x(0) = 0 \quad x'(0) = 2.$$

More room for Problem 2, if you need it.

3. (15 points) *Convolutions*

   Calculate the convolution, $f(t) * g(t)$, of the following functions

$$f(t) = e^{at}, \qquad g(t) = e^{bt}. \ (a \neq b)$$

4. (25 points) *Delta Functions*

   Solve the initial value problem

$$x'' + 2x' + x = t + \delta(t)$$

$$x(0) = 0 \quad x'(0) = 1.$$

More room for Problem 4, if you need it.

5. (10 points) *The Ratio Test*

   Use the ratio test to determine the radius of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{n+1}{3^n} x^n$$