Math 2280 - Practice Exam 3

University of Utah

Spring 2013

Name: _____

This is a 50 minute exam. Please show all your work, as a worked problem is required for full points, and partial credit may be rewarded for some work in the right direction.

1. (25 points) *An Endpoint Problem*

   The eigenvalues for this problem are all nonnegative. First, determine whether $\lambda = 0$ is an eigenvalue; then find the positive eigenvalues and associated eigenfunctions.

$$y'' + \lambda y = 0;$$

$$y'(0) = 0 \quad y'(\pi) = 0.$$

More room, if necessary, for Problem 1.

2. (10 points) *Converting to First-Order Systems*

   Transform the given differential equation into an equivalent system of first-order differential equations:

$$t^3 x^{(3)} - 2t^2 x'' + 3tx' + 5x = \ln t$$

3. (30 points) *Systems of First-Order ODEs*

   Find the general solution to the system of ODEs:

$$\mathbf{x}' = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \mathbf{x}.$$

More room, if necessary, for Problem 3.

4. (20 points) *Multiple Eigenvalues*[1]

   Find the general solution to the system of ODEs:

$$\mathbf{x}' = \begin{pmatrix} 3 & -1 \\ 1 & 5 \end{pmatrix} \mathbf{x}.$$

[1] This is a hint.

More room, if necessary, for Problem 4.

5. (15 points) *Matrix Exponentials*

   Calculate the matrix exponential e^A for the matrix:

$$\begin{pmatrix} 0 & 2 & 3 & 1 \\ 0 & 0 & 5 & 6 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

More room, if necessary, for Problem 5.