Math 2280 - Exam 3

University of Utah

Spring 2013

Name: ____

This is a 50 minute exam. Please show all your work, as a worked problem is required for full points, and partial credit may be rewarded for some work in the right direction.

1. (15 points) An RLC Circuit

For the RLC circuit pictured below:



Calculate I(t) for the values:

 $L = 10H \quad R = 20\Omega \quad C = 0.02F$

 $E(t) = 50 \sin 3t \text{ V}$

More room, if necessary, for Problem 1.

2. (15 points) An Endpoint Problem

The eigenvalues for this problem are all nonnegative. First, determine whether $\lambda = 0$ is an eigenvalue; then find the positive eigenvalues and associated eigenfunctions.

$$y'' + \lambda y = 0;$$
$$y'(-\pi) = 0 \quad y'(\pi) = 0.$$

More room, if necessary, for Problem 2.

3. (10 points) Converting to First-Order Systems

Transform the given differential equation into an equivalent system of first-order differential equations:

$$x^{(4)} + 6x'' - 3x' + x = \cos 3t$$

4. (25 points) Systems of First-Order ODEs

Find the general solution to the system of ODEs:

$$\mathbf{x}' = \begin{pmatrix} 2 & 3 \\ 2 & 1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 5 \\ -2t \end{pmatrix}.$$

More room, if necessary, for Problem 4.

Even more room, if necessary (you might need it!), for Problem 4.

5. (20 points) *Multiple Eigenvalues*¹

Find the general solution to the system of ODEs:

$$\mathbf{x}' = \left(\begin{array}{cc} 7 & 1\\ -4 & 3 \end{array}\right) \mathbf{x}.$$

¹This is a hint.

More room, if necessary, for Problem 5.

6. (15 points) Matrix Exponentials

Calculate the matrix exponential e^A for the matrix:

$$\left(\begin{array}{rrrr} 2 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & 3 & 1 \end{array}\right).$$

More room, if necessary, for Problem 6.